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Digital computer solution of electromagnetic transients in large power systems

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Title: Digital Computer Solution of Electromagnetic Transients in Large Power Systems.

APPROVED BY MEMBERS OF THE THESIS COMMITTEE

Thomas J. Killian, Chairman

Jack C. Riley

Chamberlain L. Foes

This thesis is an introduction to the solution of electromagnetic transients by a combination of the Bergeran method of characteristics and the application of trapezoidal rule of integration.

Three test examples solved by digital computer illustrate the step by step solution and computer programming.

To compare this method with the Laplace transformation technique, a test problem solved by both methods and also digital computer is illustrated. In conclusion, the advantages and disadvantages of both methods are compared.

TO THE OFFICE OF GRADUATE STUDIES:

The members of the Committee approve the thesis of Bijan Navidbakhsh presented May 18, 1973.

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May 18, 1973

DIGITAL COMPUTER SOLUTION OF ELECTROMAGNETIC

TRANSIENTS IN LARGE POWER SYSTEMS

by

BIJAN NAVIDBAKHSH

A thesis submitted in partial fulfillment of the
requirements for the degree of

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TO MY

PARENTS

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CHAPTER I

INTRODUCTION

The objective of this thesis is to present an introduction to a combination of the Bergeron method of characteristics and the trapezoidal rule of integration to the digital computer solution of transients in electric circuits. This method has been used both in Europe and here at the Bonneville Power Administration.[1, 2]

Since this method has not been generally available to students of electrical engineering, this paper discusses the method with complete details for transient and steady state problems involving both lumped and distributed parameter.

Chapter two discusses the Bergeron method applied to distributed parameters of transmission lines. The graphical solution with examples presents the general method and how it can be applied to a digital computer solution.

The first part of chapter three begins with the node equations and the trapezoidal rule of integration used in the digital computer

[1]H. Prinz and H. Dommel, "Uberspannungsberechnung in Hochspannungsnetzen", presented at the Sixth Meeting for Industrial Plant Managers, Munich, Germany, 1964.

[2]"Digital Computer Solution of Electromagnetic Transients in Single and Multiphase Networks", I.E.E.E. Transactions on Power Apparatus and Systems, Vol. Pas-88, pp. 388-399, April 1969

solution. The Bergeron method applied to distributed parameters of lossless line by digital computer is introduced; the equations and equivalent impedance network of a line are derived.

The trapezoidal rule of integration is applied for lumped parameters, the equation describing the relation of voltage and current are determined and the equivalent impedance network for each lumped parameter is obtained. Finally, the digital solution of a system with one nonlinear element is discussed. Three detailed examples are presented: the first, a lossless line with distributed parameters, the second, a circuit with lumped parameters, and the third, a system with a nonlinear parameter.

Chapter three consists of the example of a relatively simple problem by both the conventional Laplace transformation technique and the described computer transient program.

Chapter four includes the comparison of both techniques and the conclusion.

CHAPTER II

BERGERON METHOD

The method of characteristics with the aid of the trapezoidal rule of integration can be generalized in a method capable of solving transients in any network with distributed as well as lumped parameters.

In order to understand the method of characteristics, in the following paragraphs we shall discuss the Bergeron method, which will perform the character of the method of characteristics and some graphical example which enables us to understand the concept of our main objective--the digital computer solution of electromagnetic transient. The trapezoidal rule of integration will be described in the course of this paper.

In about 1930 the Bergeron method was originally devised by Louis Bergeron and O. Schnyder. This method is by no means new and it has been used as an aid for the calculation and determination of the transient phenomena in hydraulic systems. The Bergeron method has been used extensively in the field of hydraulics and the contributions to this field are well known. Recently there have been attempts to apply this method to electrical systems. The method is essentially a graphical process and this naturally will develop drawing errors. However, these errors can be eliminated by the use of a digital computer.

In this method transmission lines may be considered exactly without having the equivalent networks of lumped elements and the essential nature of the line is retained. This can be considered one of the advantages of this method. Another advantage is that complex forcing functions such as sinusoids and exponentials can be handled exactly, while in other methods these types of functions have to be approximated and simulated in different forms. The method is also capable of handling circuits with lumped, resistive, inductive, and capacitive elements. Non-linear elements cause little difficulty.

There are fundamental mathematical equations for all wave motion. There are a number of different types of solutions for these equations. One of these solutions is that the basis of the Bergeron method is such that it is exact and does not rely on mathematical series for its expression. Since transients are often discontinuous, this is an important point.

Let us now continue our discussion on relation of voltage and current on a lossless line. The following two equations describe the relation of voltage and current on a transmission line.

$$-\partial e/\partial x = L.\ \partial i/\partial t \qquad (1)$$

$$-\partial i/\partial x = C.\ \partial e/\partial t \qquad (2)$$

where:

e = voltage to ground

i = line current

t = time

L = line inductance per unit length

C = line capacitance per unit length

x = distance on the transmission line

We may introduce two notations, Z and a.

where:

Z = surge impedance of the line

a = velocity of propagation of the disturbance

Their relations with L and C are:

$$Z = \sqrt{L/C}$$

$$a = 1/\sqrt{L \cdot C}$$

By substitution of one equation for another we obtain two equations of voltage and current as follows:

$$\partial^2 i/\partial x^2 = 1/a^2 \cdot \partial^2 i/\partial t^2 \qquad (3)$$

$$\partial^2 e/\partial x^2 = 1/a^2 \cdot \partial^2 e/\partial t^2 \qquad (4)$$

The classical solution of these two equations is:

$$i = F_1\ (x-at) + F_2\ (x+at) \qquad (5)$$

$$e = Z \cdot F_1\ (x-at) - Z \cdot F_2\ (x+at) \qquad (6)$$

Here F_1 and F_2 are some functions and voltage and current are functions of time.

We can think of the expressions $(x+at)$ and $(x-at)$ as waves traveling in the negative and positive direction, respectively. Now by some manipulation of equations (5) and (6) we will see the principle of Bergeron method clearly.

Let us multiply equation (5) by Z and first add it, then subtract it from equation (6). The result will be the two following equations:

$$e + Z \cdot i = 2Z \cdot F_1(x-at) \qquad (7)$$

$$e - Z \cdot i = -2Z \cdot F_2(x+at) \qquad (8)$$

If the two expressions (x-at) and (x+at) in equation (7) and (8) are constant, the left hand side of the two equations will also be constant.

Now if we think of an observer who travels along the line with a constant velocity "a" in the positive x direction, then this observer would tell us that the expression $e + Z.i$ appears constant to him.

If we express this in physical terms, we may understand the concept of two equations (7) and (8) thoroughly. This simply can be expressed so that if we think of an observer who is able to travel along the line with the traveling wave, at the same speed as the wave is moving, and at the same time is able to measure the voltage and current flowing, he would see that the relation of voltage and current is $e + Z.i = \text{constant}$.

The result of the above discussion permits us to write equations (7) and (8) as the following forms:

$$e + Z \cdot i = \text{Constant} \qquad (9)$$

$$e - Z \cdot i = \text{Constant} \qquad (10)$$

Equations (9) and (10) are the equations of two straight lines in the voltage and current plane with the slopes of +Z and -Z, respectively. This is the basic idea of the Bergeron method.

In order to formulate the principle of this method we have to express another point.

Let us assume two observers are moving along a transmission line, one in positive and the other in negative direction. As they are moving along the line they meet each other at a point where the voltage and current that one observer is measuring is the same as the other.

Equations (9) and (10) are called the "characteristics" of the preceeding differential equations. From this we may say that the Bergeron method is the extension of the "Method of Characteristic".

In order to see the practical aspect of this method we illustrate some examples as follows:

Distributed-Constant Transmission Line

Consider the distributed-constant transmission line AC and point B between A and C as shown in fig. (1)a

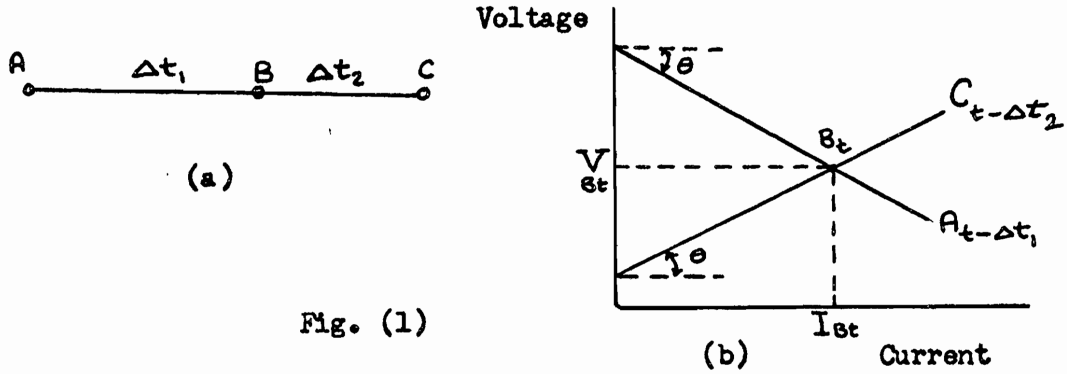


Fig. (1)

Δt_1 and Δt_2 represent the transit time for sections AB and BC, respectively. This means Δt_1 unit of time takes the wave travel from A to B and Δt_2 unit of time from B to C.

Now if some observer at time $(t-\Delta t_1)$ starts traveling from point A toward point B with wave velocity, he would see the relation of voltage and current along the line as follows:

$$e + Z.i = K_1 \tag{11}$$

Here K_1 is constant.

If another observer starts traveling from point C toward point B with wave velocity, he would see the relation of voltage and current along the line as follows:

$$e - Z.i = K_2 \tag{12}$$

Suppose by some means the voltage and current at point A at time $(t-\Delta t_1)$ and at point C at time $(t-\Delta t_2)$ are known. By having this information equations (11) and (12) can be solved for K (constant) and consequently the points and lines $A_{(t-\Delta t_1)}$ and $C_{(t-\Delta t_2)}$ can be plotted in voltage and current plane as shown in fig. (1)b. The slopes of the lines $A_{(t-\Delta t_1)}$ and $C_{(t-\Delta t_2)}$ are $-Z$ and $+Z$, respectively.

Now if we go back to our observers, we see that they will meet each other at point B at time t and both register the same value for voltage and current. Consequently, by having the past history we are able to determine voltage and current at any point and at any instant of time.

Transmission Line with a Non-Linear Resistor

Let us turn our attention to a more complicated and practical example. Consider the system shown in fig. (2). There is a transmission line terminated by a non-linear resistor. The characteristic of the resistor and the parameters of the line are known. For this example the line is considered to be lossless.

The non linearity of an element in a system causes some complexity as far as the Bergeron method is concerned. The transient voltage across the resistor and the transient current at the source are desired while a step voltage is applied at the point A at time zero.

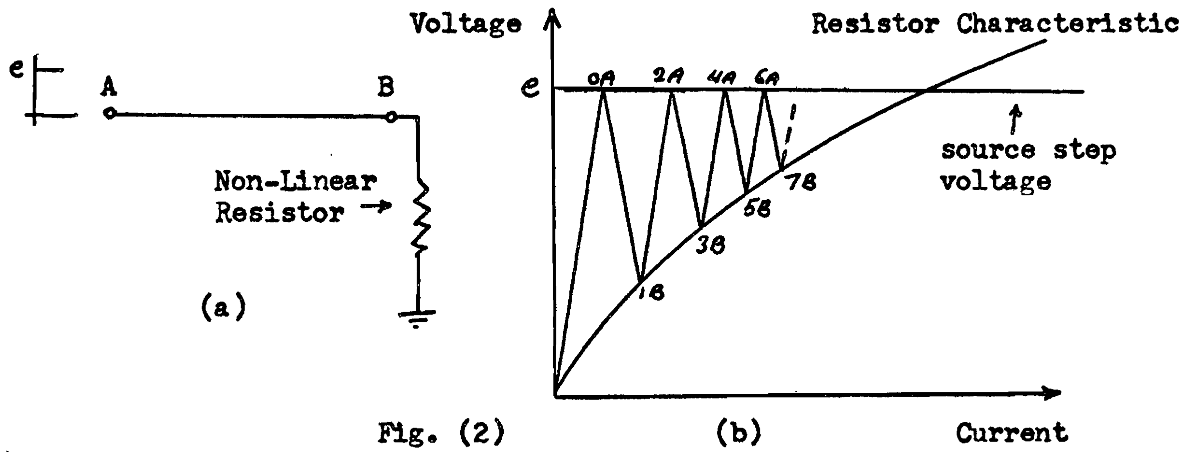


Fig. (2) (b)

One very important advantage of the Bergeron method is that it enables us to compute current and voltage simultaneously. In spite of other methods there is no need for separate computation to determine the transient current. This advantage will appear in the digital computer solution of electromagnetic transient which shall be discussed in full details in further sections. The Bergeron diagram for this system is shown in fig. (2)b.

Let us consider an observer moving at the wave speed up and down along the transmission line. Every time this observer travels the distance AB or BA it takes him one time unit. Here the time unit corresponds to the transit time of the line. Suppose he is leaving the receiving end B at a time -1. This means one time unit before the step voltage is applied at A. We shall notice in the following paragraph that it is necessary to have some information of the past history in a short period of transit time.

Consider equations (11) and (12). Every time the observer travels the distance AB and BA we should be able to show the straight lines corresponding equations (11) and (12), respectively. He will reach A at time zero just as the step is applied. We know voltage and current at time -i at B is zero. From this equation (11) becomes a straight line with the form of $e = Zi$ passing the origin and intersecting the step voltage e at point oA in Bergeron diagram at time zero. The slope of this line is +Z where Z is the line surge impedance. This line is represented by o, oA as shown in fig. (2)b. Now at point oA (o represents time zero and A represents terminal A in fig. (2)a) we know voltage and current at time zero. In the next step when the observer travels from A to B we know already the past history for the next step time +1. The equation describes the relation of voltage and current at this time is equation (12). This is a line passing through point oA with the slope of -Z and intersecting the resistor characteristic at point 1B. The characteristic of his travel from time -1 until time +1 is the line o, oA and oA, 1B.

The intersection of the line oA, 1B with a line representing the characteristics of the resistor tells us the required voltage and current at B at time +1. We may notice that the characteristics of the resistor can take any degree of complexity without really making the problem more complicated.

To continue the solution of this problem for further steps, the observer simply continues his travel back and forth along the transmission line and the diagram is built up as shown in fig. (2)b.

Lumped Elements

Since the Bergeron method is not our main objective and it is only to give us a basic background for our further discussion in chapter two, we limit ourselves to a brief description of the handling of lumped elements in the Bergeron method.

Important elements in power systems are of course lumped inductive and capacitive components. There must be some methods which are able to handle lumped elements in power systems. One of these methods is presented by the Bergeron method. The one possible disadvantage in this case is that the results must be a little approximated. The approximation taken in this method is quite controllable and the result obtained by the Bergeron method is not worse than those from other processes.

To show the process of the method a lumped inductance example is illustrated as follows:

Lumped Inductance

Consider a transmission line terminated by a lumped inductance as shown in fig. (3)a. Let us assume some disturbance is applied at A and the voltage and current at B is known at time t. Since we know the required information of B at time t, the point B_t can be plotted in fig. (3)b.

The voltage across an inductance is given by:

$$e = L \cdot di/dt \qquad (13)$$

This will be handled by using finite differences rather than differentials, therefore let $dt = h$.

We want to find the voltage at point B at time $t + h$. The point B_{t+h} must be on the straight line characteristic $A_{t+h-\Delta t}$, B seen by an observer leaving A at time $t + h - \Delta t$, where Δt is the transit time from A to B. Now if h is small enough that we can assume e varies linearly in this interval, e_t and e_{t+h} are the voltages at point B at time t and t+h respectively, then we may write equation (13) in the form of:

$$((e_t + e_{t+h})/2) \cdot h = L \cdot di \qquad (14)$$

or

$$e_{t+h} = -e_t + L/(h/2) \cdot di \qquad (15)$$

Equation (15) is plotted in fig. (3)b represented by the line B_t', S. The slope of this line is $L/(h/2)$. This means $\tan \theta = L/(h/2)$.

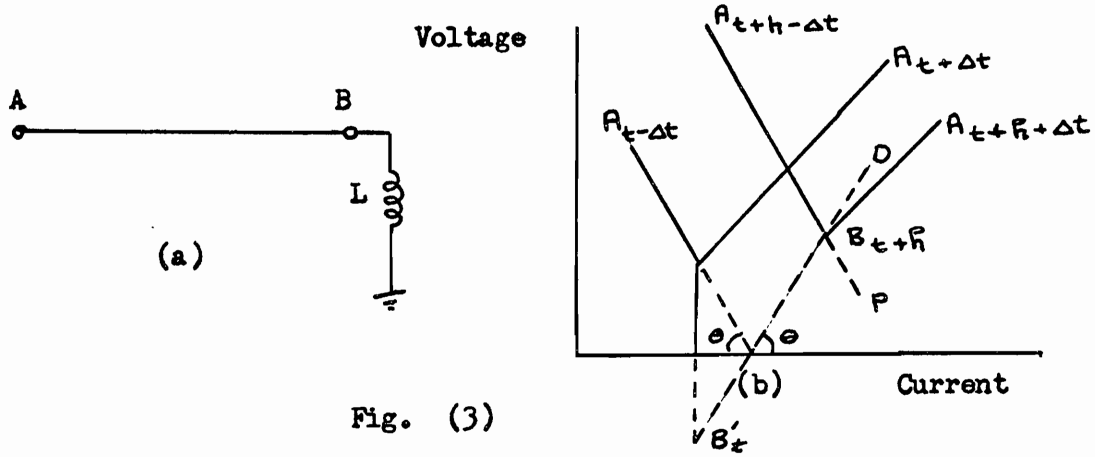


Fig. (3)

Since we know the angle θ, we can plot the equation of the line B_t',S and since the point B_t' is symmetrical with B_t, the line B_t', S can be plotted. This line intersects the line $A_{t+h-\Delta t}$, P at the desired point B_{t+h}.

The complete diagram is built up by assuming that an observer travels back and forth along the transmission line AB. The surge impedance of such a line is L/(h/2) and transit time is h/2. In this case it might be argued that the inductor has been simulated by a transmission line.

Lumped Capacitance and Series Lumped Circuit

The same process could be done in a case of lumped capacitance. In this case the surge impedance is h/2c and the line transit is h/2.

The series lumped circuit can be handled in the Bergeron method by the method just described, which may be "Cascaded" and this enables us to solve any combination of such elements in any electrical circuit.

The Bergeron method provides a powerful tool for the solution of transient voltage and current in electrical engineering. So far this method taught us how to determine the transient voltage and current graphically in an electrical circuit. The concept of this method will be applied to our next discussion regarding the digital computer solution.

CHAPTER III

DIGITAL COMPUTER TECHNIQUE

The computation of the transient voltage and current was discussed by means of the graphical method in the previous chapter. As mentioned before, the disadvantage of this graphical method is the errors made by drawing and the time consumed for the solution of a large power system. However, graphical errors and the problem of time can be eliminated by means of digital computation.*

The important digital computer solutions which are discussed in this chapter are the method of characteristics and the trapezoidal rule of integration. Since node equations play an important role in digital computer solution, these are discussed first and then the trapezoidal rule of integration is presented.

Node Equations

The system of node equations, applying the Kirchhoff's current law, is an important fundamental base for a digital computer solution of a large power system. In order to derive a general formula for node equations in any electrical circuit, consider the circuit shown in fig. (4). The Kirchhoff's current law states that the sum of the currents at each node is zero.

*Of course, the main purposes of using digital computers are automatic solution and faster facility in obtaining data.

Applying this law at node 1, 2, 3, and 4 with direction of current as shown in fig. (4) (the arrow represents the direction of current), gives

$$y_g(e - V_1) = Y_a(V_1 - V_2) + Y_c(V_1 - V_4) \qquad (16)$$

$$0 = Y_a(V_1 - V_2) - Y_b(V_2 - V_3) - Y_e(V_2 - V_4) \qquad (17)$$

$$0 = Y_b(V_2 - V_3) - Y_m(V_3 - 0) - Y_d(V_3 - V_4) \qquad (18)$$

and for node 4

$$0 = -Y_p(V_4 - 0) + Y_d(V_3 - V_4) + Y_c(V_1 - V_4). \qquad (19)$$

Rearranging this equation and letting $Y_g e = I_1$ gives

$$I_1 = V_1(Y_g + Y_a + Y_c) - V_2Y_a - V_4Y_c \qquad (20)$$

$$0 = V_1Y_a - V_2(Y_a + Y_b + Y_e) + V_3Y_b + V_4Y_e \qquad (21)$$

$$0 = V_2Y_b - V_3(Y_b + Y_m + Y_d) + V_4Y_d \qquad (22)$$

$$0 = V_1Y_c + V_3Y_d - V_4(Y_p + Y_d + Y_c). \qquad (23)$$

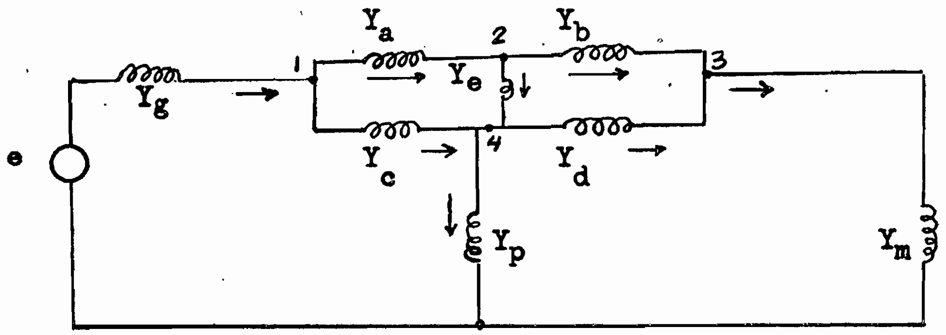


Fig. (4)

The node voltages V_1, V_2, V_3, and V_4 can be determined by solving the four equations 20 - 23 simultaneously. By knowing these voltages, all branch currents can be found. We should note that the number of equations is one less than the number of nodes.

From the above four equations we can derive the general node equation for any electrical circuit as follows:

$$I_1 = V_1Y_{11} + V_2Y_{12} + V_3Y_{13} + \ldots\ldots\ V_nY_{1n}$$

$$I_2 = V_1Y_{21} + V_2Y_{22} + V_3Y_{23} + \ldots\ldots\ V_nY_{2n}$$

$$I_3 = V_1Y_{31} + V_2Y_{32} + V_3Y_{33} + \ldots\ldots\ V_nY_{3n}$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$I_n = V_1Y_{n1} + V_2Y_{n2} + V_3Y_{n3} + \ldots\ldots\ V_nY_{nn}$$

In matrix form it is written as:

$$[I] = [Y]\,[V]$$

or

$$[V] = [Y]^{-1}[I]$$

where, in our case $n = 4$ and I_2, I_3, and I_4 are zero.

Trapezoidal Rule of Integration

There are a number of ways to integrate a function and one of these is the trapezoidal method which is used in many digital computer solution techniques. In order to see the rule of this integration method, consider function F in Fig. (5) (F could be any continuous function).

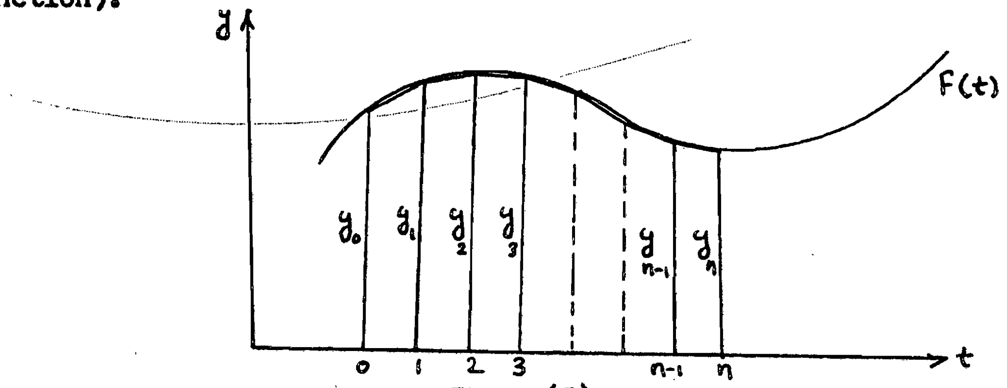


Fig. (5)

The t axis is equally divided in n small division and each small division is designated by Δt. The area below the curve between zero and n determines the integral of function F from 0 to n. Let us write the area of each trapezoid under the curve and then by summation of all areas, the integral of F will be found.

$$A_1 = (1/2)(y_0 + y_1) \Delta t$$

$$A_2 = (1/2)(y_1 + y_2) \Delta t$$

$$\cdots\cdots\cdots\cdots$$

$$\cdots\cdots\cdots\cdots$$

$$A_{n-1} = (1/2)(y_{n-2} + y_{n-1}) \Delta t$$

$$A_n = (1/2)(y_{n-1} + y_n) \Delta t.$$

By adding all areas the total area will be:

$$A_t = (\Delta t/2)(y_0 + 2y_1 + 2y_2 + \ldots + 2y_{n-1} + y_n) = \int_0^n F dt$$

Of course one could say this result may not be accurate enough since an approximation is used. This will depend on how small t is. The use of Δt will be made more clear in the next part where the digital computer solution is discussed.

DIGITAL COMPUTER SOLUTION OF ELECTOMAGNETIC TRANSIENTS

A digital computer solution of transients can be applied to power systems containing long lines with or without lumped parameters. The process taken in this digital computer solution is a step by step procedure that proceeds along the time axis where each step is designated by Δt. This short interval of time may be either fixed or variable. Starting with initial condition $t = 0$, the state of the system in the sequence of steps is found at $t = \Delta t$, $2\Delta t$, $3\Delta t$, ... up to the maximum time t_{max} for a particular case. In every state of the process the past history is known. This means when we are solving for time t the previous states $t - \Delta t$, $t - 2\Delta t$, $t - 3\Delta t$, and back to the initial conditions are known. In the case of long lines the past history used for time t is $t - \tau$ where τ corresponds to the travel time of the line. In the case of lumped parameters Δt corresponds only to the previous step where it can be any arbitrary length of time. Therefore a limited portion of this "past history" is used for long lines in the method of characteristics and in the trapezoidal rule of integration.

Equivalent impedance network of a line and lumped parameters are presented by the equations derived by the method of characteristics for lines and trapezoidal rule of integration for lumped parameters and the record of past history. This will be made clearer as we first discuss lossless lines and the lumped parameter networks.

Lossless Line

In order to derive an equation by the method of characteristics for lossless line and consequently to build an equivalent impedance network for the line, we have to remind the reader of some material which has been already discussed in chapter two. This also helps us to see that the digital computer solution is actually Bergeron's method for a lossless line.

Consider a lossless line with inductance L and capacitance C per unit length (fig. 6). The relation of voltage and current at a point x along the line is

$$-\partial e/\partial x = L(\partial i/\partial t) \tag{24}$$

$$-\partial i/\partial x = C(\partial e/\partial t) \tag{25}$$

The general solution, first given by d'Alembert, is:

$$i(x,t) = F_1(x-at) + F_2(x+at) \tag{26}$$

$$e(x,t) = Z.F_1(x-at) - Z.F_2(x+at) \tag{27}$$

where

$$Z = \sqrt{L/C}$$

$$a = 1/\sqrt{LC}$$

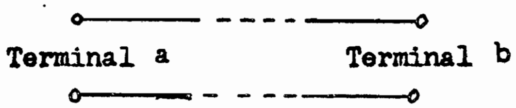


Fig. (6) Lossless Line

and by some manipulation which was discussed in chapter two, equations (26) and 27) can be written in the form of:

$$e(x,t) + Z.i(x,t) = 2Z.F_1(x - at) = \text{constant} \tag{28}$$

$$e(x,t) - Z.i(x,t) = -2Z.F_2(x + at) = \text{constant.} \tag{29}$$

As we see equations (24) - (29) are the same as equations (1) - (8) in the previous chapter.

Now with the same imaginary observer as before, who travels along the line, equations (28) and (29) seen by the observer are constant. This means when he travels from b at time $t - \tau$ equation (28) is equal to equation (29) when he reaches a at time t. If the travel time to get from b to a or a to b is

$$\tau = 1/a$$

(1 is the length of the line), then with the same logic we have:

$$e_a(t) - Zi_{a,b}(t) = e_b(t - \tau) + Zi_{b,a}(t - \tau) \quad (30)$$

$$e_b(t) - Zi_{b,a}(t) = e_a(t - \tau) + Zi_{a,b}(t - \tau) \quad (31)$$

$i_{a,b}$ and $i_{b,a}$ can be found from equations (30) and (31) as follows:

$$i_{a,b}(t) = (1/Z)e_a(t) + (-(1/Z)e_b(t - \tau) - i_{a,b}(t - \tau)) \quad (32)$$

$$i_{b,a}(t) = (1/Z)e_b(t) + (-(1/Z)e_a(t - \tau) - i_{a,b}(t - \tau)). \quad (33)$$

Consider equation (32); the second term of the right hand side of this equation can be simulated as a source current parallel with Z and let us call this current $I_a(t-\tau)$. Therefore equation (32) can be written in the following form:

$$i_{a,b}(t) = (1/Z)e_a(t) + I_a(t - \tau) \quad (34)$$

where

$$I_a(t - \tau) = -(1/Z)e_b(t - \tau) - i_{b,a}(t - \tau). \quad (35)$$

Consequently the equivalent impedance network by considering equations (34) for node a is illustrated in fig. (7).

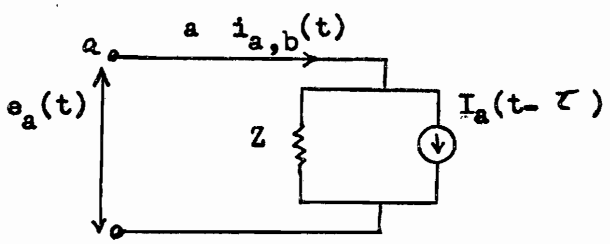


Fig. (7)

Similarly, for node b equation (33) can be written in the form of:

$$i_{b,a}(t) = (1/Z)e_b(t) + I_b(t - \tau) \quad (36)$$

where

$$I_b(t - \tau) = -(1/Z)e_a(t - \tau) - i_{a,b}(t - \tau) \quad (37)$$

and the equivalent impedance network considering equation (36) for node b is shown in fig. (8)

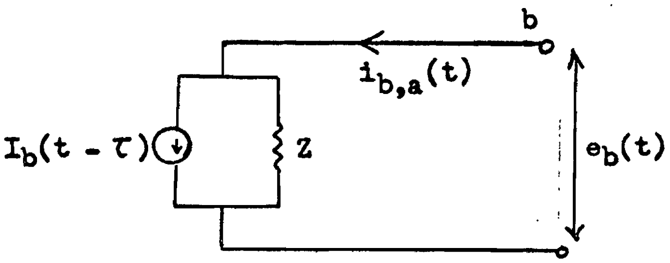


Fig. (8)

Now, if fig. (7) and (8) are combined in an imaginary box, the complete equivalent impedance network of the line is a two port network as shown in fig. (9).

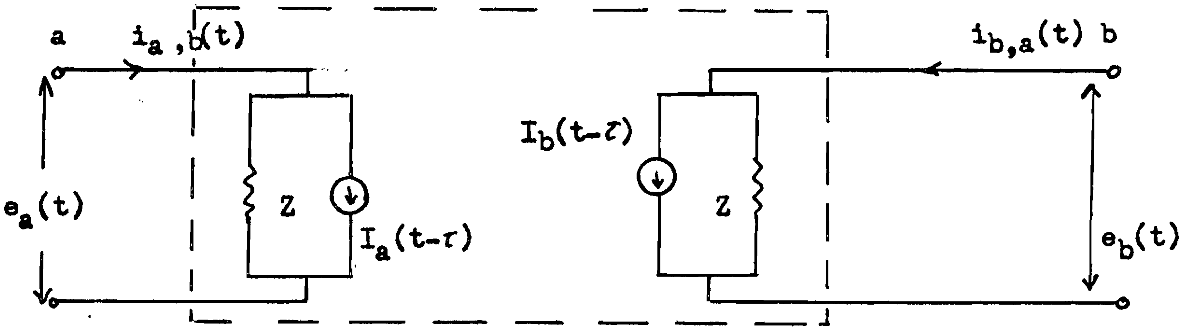


Fig. (9) Equivalent impedance network

This fully describes the lossless line at its terminals. As the diagram shows, the terminals are not connected; of course the conditions at the other terminal in respect to another are only seen indirectly by means of equations (30) - (33) with a time delay .

Although the method of characteristics can handle lossy lines also, the differential equations produced are not directly integrable. Therefore, losses are neglected at this stage. They may be included later as equivalent lumped resistances.

In order to indicate the complete procedure a digital computer solution for a lossless line is illustrated in full detail as follows:

Test Case No. 1

Consider a series of three different lines, which are terminated by an infinite line. A rectangular voltage impulse, coming from an infinite line, is applied to these lines. This is shown in fig. (10). The voltage at node 4 is desired for maximum time of 20 micro-seconds.*

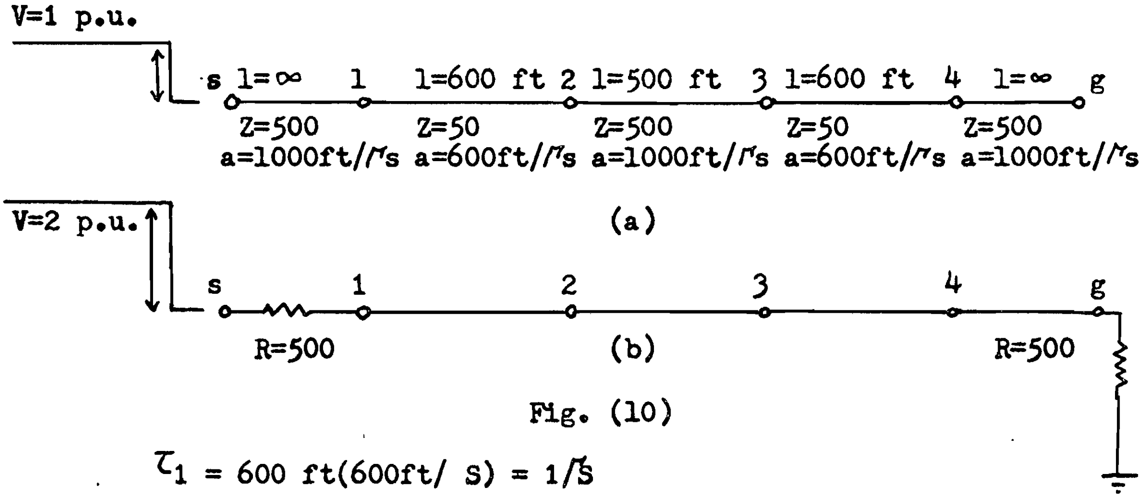


Fig. (10)

$$\tau_1 = 600\text{ ft}(600\text{ft}/\ \text{s}) = 1\mu\text{s}$$

$$\tau_2 = 500\text{ ft}/(1000\text{ft}/\ \text{s}) = .5\ \text{s}$$

$$\tau_3 = 600\text{ ft}/(600\text{ft}/\ \text{s}) = 1\mu\text{s}$$

Infinite line means the travel time on the line is more than the time of study and when a voltage impulse of 1 p.u. is coming over this line, it is represented by voltage impulse of 2 p.u. coming over its

*This problem is taken from L. V. Bewley, Traveling Waves on Transmission Systems, (New York: Dover Publications, Inc., 1951), p. 100.

equivalent resistance R = Z. Infinte line 4, g shown in fig. (10)a is represented as a resistance to ground with R = Z as shown in Fig. (10)b.

Given data:

$$\text{Excitation:} \quad e(t) = \begin{cases} 0 & t < 0 \\ 2.0 & t > 0 \end{cases}$$

$$\Delta t = .25\,\mu s$$

$$t_{max.} = 20\,\mu s$$

Solution:

The first step is to draw the equivalent network impedance of the system. This could be done by using fig. (9) for each node and the second step is to write node equations for all nodes. For this type of problem (lossless lines) there is no need to put node equations in matrix form, because all equations can be solved independently. (The equivalent impedance network of the system is completely shown in fig. (11).

Node Equations:

Consider each node in fig. (11); the node equations for node 1, 2, 3, and 4 are found as follows:

$$i_{1,2} - i_1(t) = (1/50)e_1 + I_1'(t - \tau_1) - (2 - e_1)/500 = 0 \qquad (38)$$

$$i_{2,1} + i_{2,3} = (1/50)e_2 + I_2(t - \tau_1) + (1/500)e_2 + I_2'(t - \tau_2) = 0 \quad (39)$$

$$i_{3,2} + i_{3,4} = (1/500)e_3 + I_3(t - \tau_2) + (1/50)e_3 + I_3'(t - \tau_3) = 0 \quad (40)$$

$$i_{3,4} + i_4 = (1/50)e_4 + I_4(t - \tau_3) + (1/500)e_4 = 0 \qquad (41)$$

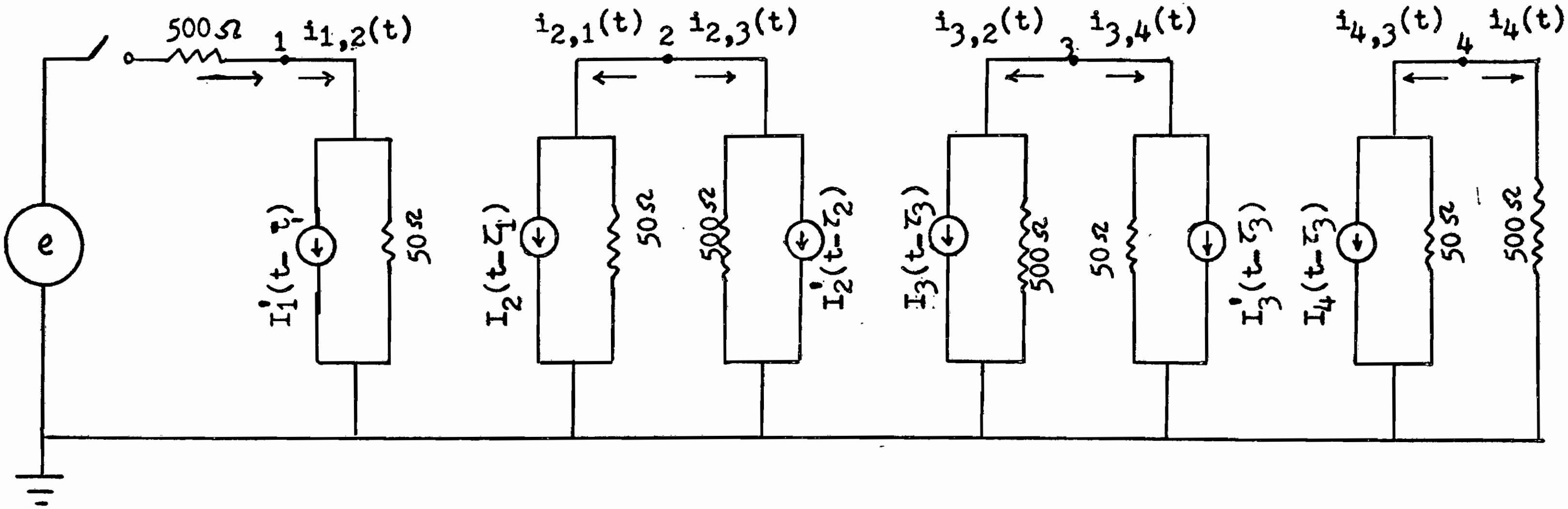


Fig. (11) Equivalent Impedance Network

Node voltages e_1, e_2, e_3, and e_4, can be found from equations (38)-(41) in the following form:

$$e_1(t) = (500/11)\ (2/500 - I_1'(t - \tau_1)) \tag{42}$$

$$e_2(t) = (500/11)\ (-I_2(t - \tau_1) - I_2'(t - \tau_2)) \tag{43}$$

$$e_3(t) = (500/11)\ (-I_3(t - \tau_2) - I_3'(t - \tau_3)) \tag{44}$$

$$e_4(t) = (500/11)\ (-I_4(t - \tau_3)) \tag{45}$$

In order to compute the node voltages, we need to compute I and I'. By considering equations (32) - (35), we have:

$$i_{2,1}\ (t) = (1/50)e_2(t) + I_2(t - \tau_1)$$

$$I_1'(t - t_1) = -\ (1/50)e_2(t - \tau_1) - i_{2,1}(t - \tau_1)$$

$$i_{1,2}(t) = (1/50)e_1(t) + I_1'(t - \tau_1)$$

$$I_2(t - t_1) = -\ (1/50)e_1(t - \tau_1) - i_{1,2}(t - \tau_1)$$

$$i_{3,2}(t) = (1/500)e_3(t) + I_3(t - \tau_2)$$

$$I_2'(t - \tau_2) = -(i/500)e_3(t - \tau_2) - i_{3,2}(t - \tau_2)$$

$$i_{2,3}(t) - (1/500)e_2(t) + I_2'(t - \tau_2)$$

$$I_3(t - \tau_2) + -(1/500)e_2(t - \tau_2) - i_{2,3}(t - \tau_2)$$

$$i_{4,3}(t) = (1/50)e_4(t) + I_4(t - \tau_3)$$

$$I_3'(t - \tau_3) = -(1/50)e_4(t - \tau_3) - i_{4,3}(t - \tau_3)$$

$$i_{3,4}(t) = (1/50)e_3(t) + I_3'(t - t_3)$$

$$I_4(t - \tau_3) = -(1/50)e_3(t - \tau_3) - i_{3,4}(t - \tau_3)$$

Now we have all the information to write a digital computer program for this system.

Digital Computer Program:

Before we write a program, we have to describe some notations used in the program. These notations correspond to the previous notations and they are as follows:

T1 = maximum time

D = Δ t

I = ($T_{max.}$/ Δ t) = units of time

T = time

K = $\tau_1 = \tau_3$ (in unit of time)

N = τ_2 (in unit of time)

E(n,I) = $e_n(t)$ n = 1, 2, 3, 4

R (2,I) = $i_{2,1}(t)$

R(1,I) = $i_{1,2}$ (t)

P(2,I) = $i_{2,3}(t)$

The program written on the next page is in basic language and has been executed by G. E. Mark II.

```
100 DIM E(4,100),R(4,100),A(3,100),B(4,100),P(3,100)
110 READ T1,D
120 READ E(1,1),A(1,1),A(2,1),B(2,1),E(3,1),E(2,1)
130 READ A(3,1),B(3,1),E(4,1),B(4,1)
140 PRINT "TIME","VOLTAGE AT NODE 4"
150 J=1+T1/D
160 FOR I=2 TO J
170 T=D*(I-1)
180 K=I-4
190 N=I-2
200 IF T> .75 THEN 210
204 K=1
206 N=1
210 E(1,I)= (500/11)*(2/500-A(1,K))
215 E(2,I)=(500/11)*(-B(2,K)-A(2,N))
220 R(2,I)=(1/50)*E(2,I)+B(2,K)
230 A(1,I)=-(1/50)*E(2,I)-R(2,I)
240 R(1,I)=(1/50)*E(1,I)+A(1,K)
250 B(2,I)=-(1/50)*E(1,I)-R(1,I)
270 E(3,I)=(500/11)*(-B(3,N)-A(3,K))
280 R(3,I)=(1/500)*E(3,I)+B(3,N)
290 A(2,I)=-(1/500)*E(3,I)-R(3,I)
300 P(2,I)=(1/500)*E(2,I)+A(2,N)
310 B(3,I)=-(1/500)*E(2,I)-P(2,I)
330 E(4,I)=-(500/11)*B(4,K)
340 R(4,I)=(1/50)*E(4,I)+B(4,K)
```

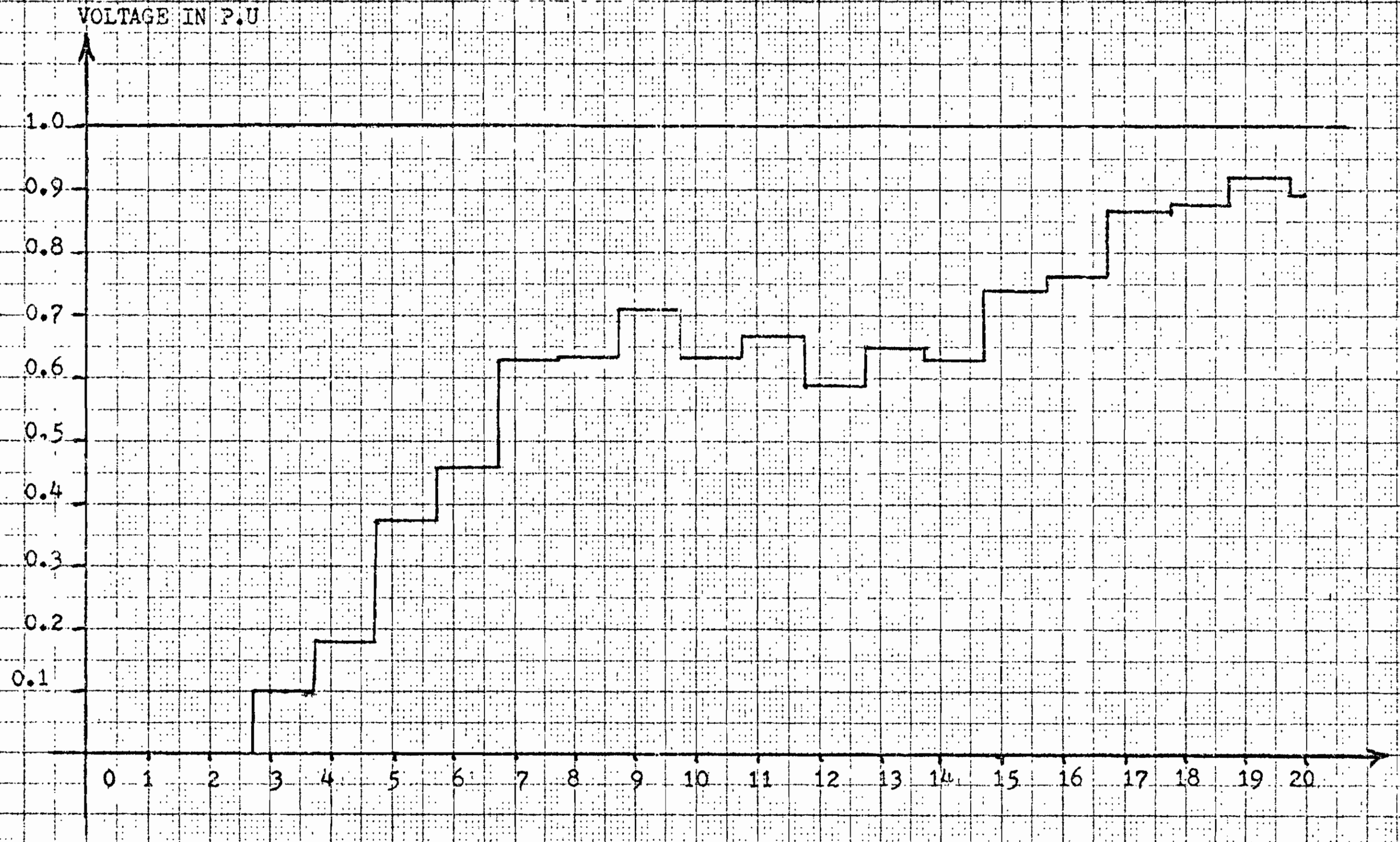
```
350 A(3,I)=-(1/50)*E(4,I)-R(4,I)
360 P(3,I)= (1/50)*E(3,I)+A(3,K)
370 B(4,I)=-(1/50)*E(3,I)-P(3,I)
380 PRINT T,E(4,I)
390 NEXT I
400 DATA 20,.25
410 DATA .0,.0,.0,.0,.0,.0
420 DATA .0,.0,.0,.0
999 END
```

The result is printed on the next page.

TIME	VOLTAGE AT NODE 4
0.25	0
0.5	0
0.75	0
1	0
1.25	0
1.5	0
1.75	0
2	0
2.25	0
2.5	0
2.75	0.109282
3	0.109282
3.25	0.109282
3.5	0.109282
3.75	0.182438
4	0.182438
4.25	0.182438
4.5	0.182438
4.75	0.377722
5	0.377722
5.25	0.377722
5.5	0.377722
5.75	0.460081
6	0.460081
6.25	0.460081
6.5	0.460081
6.75	0.629753
7	0.629753
7.25	0.629753
7.5	0.629753
7.75	0.63252
8	0.63252
8.25	0.63252
8.5	0.63252
8.75	0.712729
9	0.712729
9.25	0.712729
9.5	0.712729
9.75	0.636784
10	0.636784
10.25	0.636784
10.5	0.636784
10.75	0.668727
11	0.668727
11.25	0.668727
11.5	0.668727
11.75	0.593386
12	0.593386

12.25	0.593386
12.5	0.593386
12.75	0.653305
13	0.653305
13.25	0.653305
13.5	0.653305
13.75	0.637505
14	0.637505
14.25	0.637505
14.5	0.637505
14.75	0.742868
15	0.742868
15.25	0.742868
15.5	0.742868
15.75	0.76717
16	0.76717
16.25	0.76717
16.5	0.76717
16.75	0.868368
17	0.868368
17.25	0.868368
17.5	0.868368
17.75	0.874778
18	0.874778
18.25	0.874778
18.5	0.874778
18.75	0.924228
19	0.924228
19.25	0.924228
19.5	0.924228
19.75	0.890065
20	0.890065

VOLTAGE AT NODE 4



RESULT OF TEST CASE NO. 1

In the previous example a digital computer solution for a lossless line was fully described. In order to see that different types of systems with a lossless line are terminated in a resistor, inductor, or capacitor, some examples of traveling wave on single phase lines will be illustrated.

The following examples are taken from H. Prinz, W. Zaengl and O. Volaker.*

In most examples, the line is assumed to be lossless and the surge impedance for a single phase line with a $R' =$ and $G' = 0$ is:

$$Z = \sqrt{L'/C'}$$

and travel time is:

$$\tau = l \cdot \sqrt{L'C'}$$

where

l = length of line

L' = Series inductance per unit length

C' = Shunt capacitance per unit length

1. Line Terminated in R (R > Z)

Energization is from dc source. The result is shown in fig. (12), a, b, c, and d.

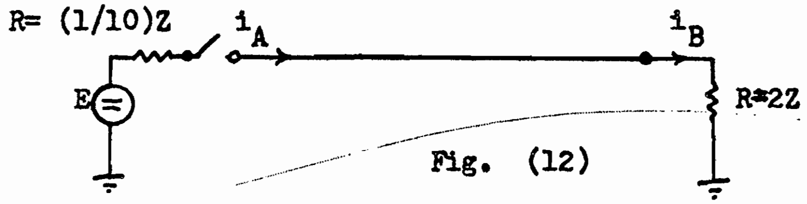
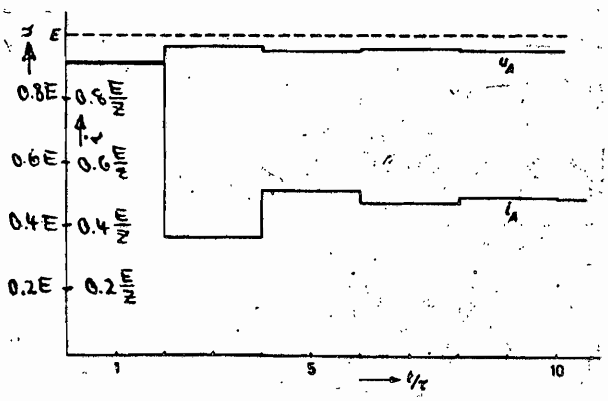
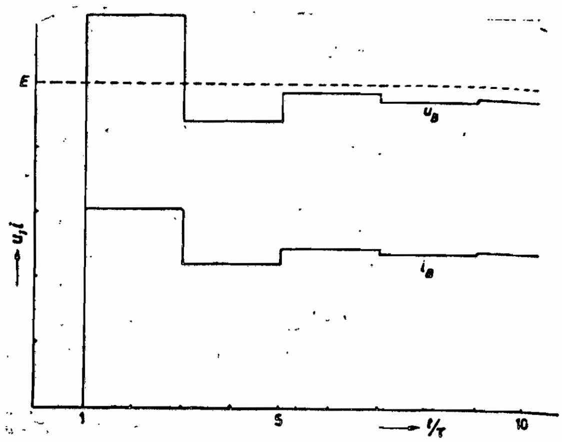


Fig. (12)

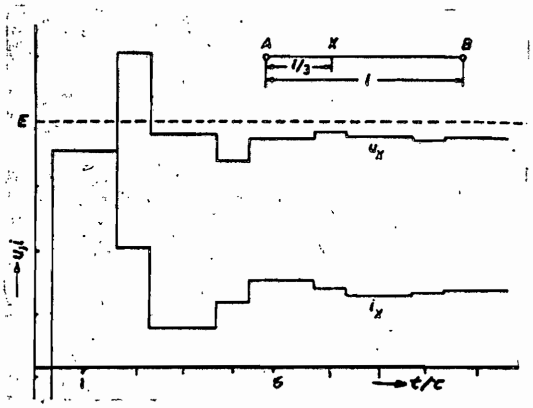
*H. Prinz, W. Zaengl and O. Volaker, "The Method of Bergeron for Solving Traveling Wave Problems" (in German), Bulletin SEV, Vol. 53, (Swiss Association of Electrical Engineering, August, 1962), pp. 725-739.



Voltage u_A and current i_A at sending end.



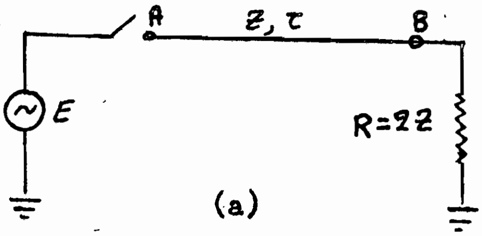
Voltage u_B and current i_B at receiving end.



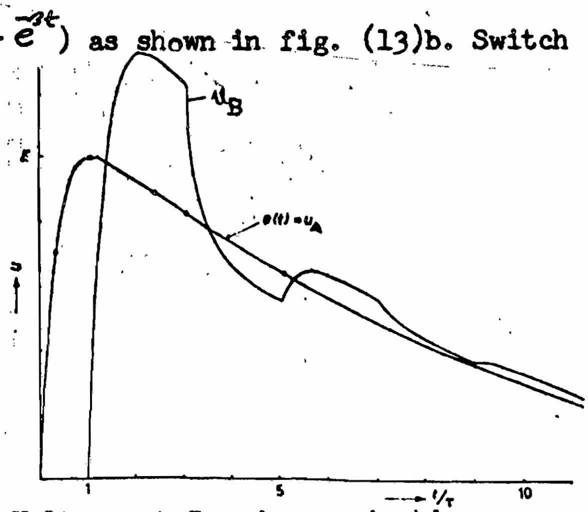
Voltage u_X and current i_X at 1/3 down the line

2. Line terminated in R (R > Z)

Energization is $e(t) = K(e^{-\alpha t} - e^{-\beta t})$ as shown in fig. (13)b. Switch closes at t = 0.



(a)

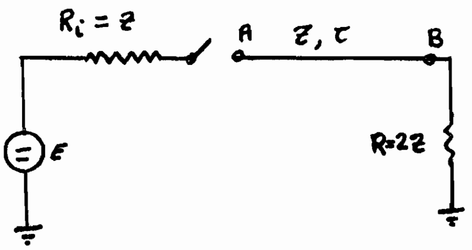


Voltage at B and energization source

(Fig. (13)) (b)

3. Line terminated in R (R>Z), internal resistance R_i = Z

Energization is from dc source.



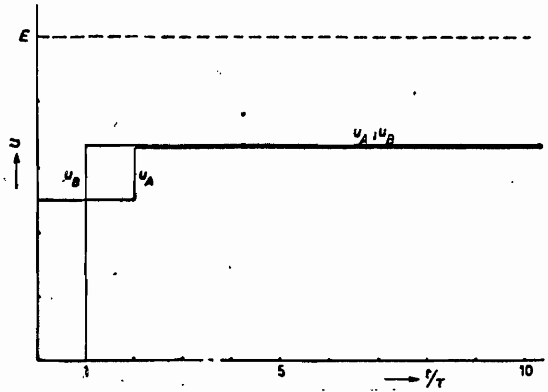
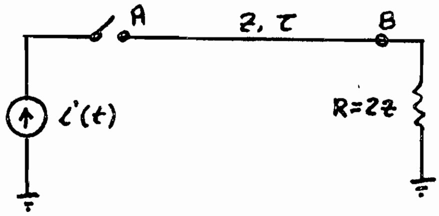


Fig. (14)

Voltage at A and B

4. Line terminated in R (R>Z), energization from current source

Switch closes at t=0, i(t)=o for t<0, i(t)=I for t$\leqslant$0



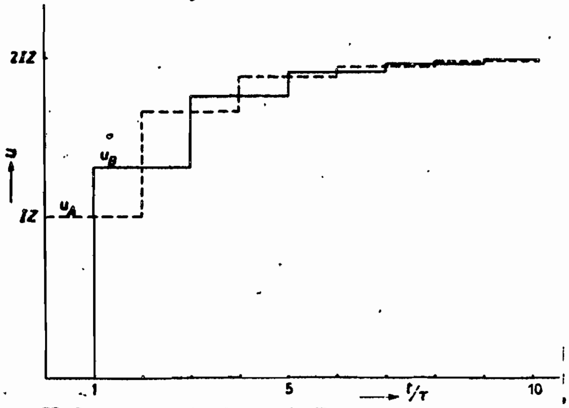
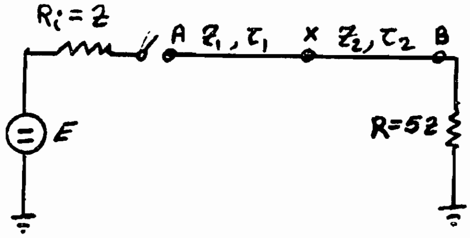


Fig. (15)

Voltage at A and B

5. Series connection of lines with unequal surge impedance

Switch closes at t=0, Z_1=(1/10)Z_2, τ_1=2τ_2.



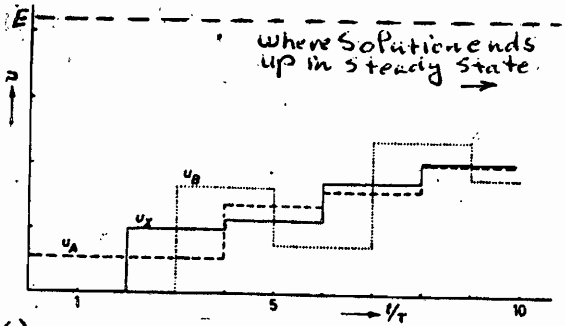


Fig. (16)

6. Reflections at line junctions

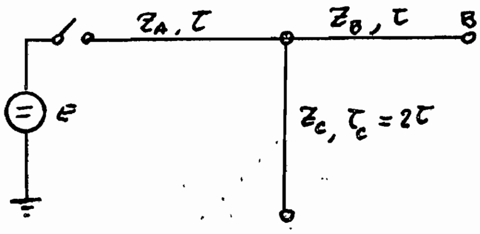
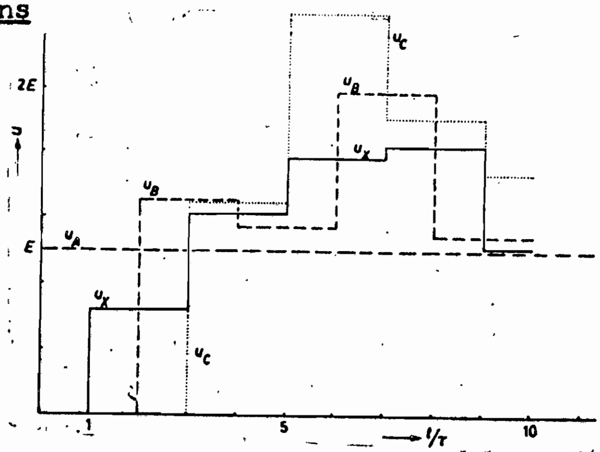


Fig. (17)



Voltage at A, B, C, and X

7. Lumped resistance in series with line

u_R is voltage across resistance, $Z_B = Z$, and $\tau_B = 2\tau$

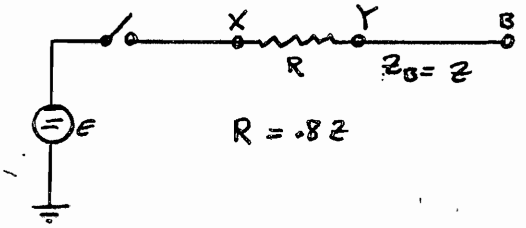
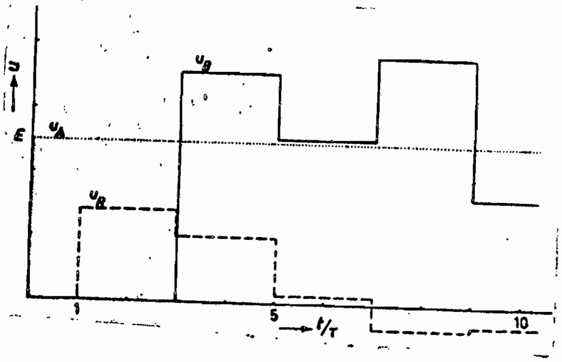


Fig. (18)



Voltage at A, B, and u_R

8. Lumped shunt resistance

R = 2Z

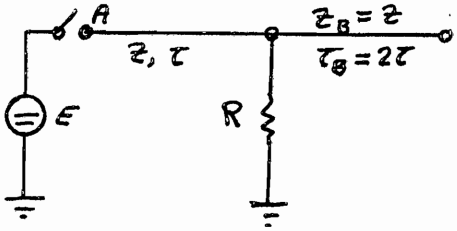
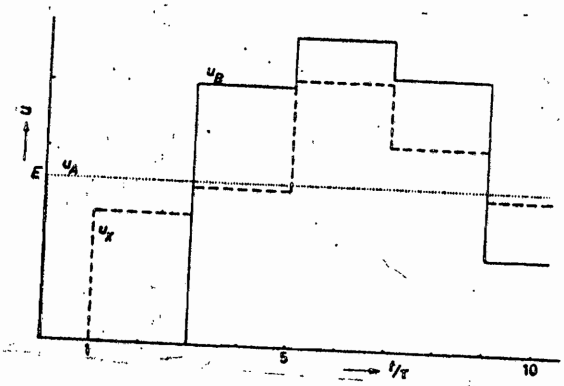


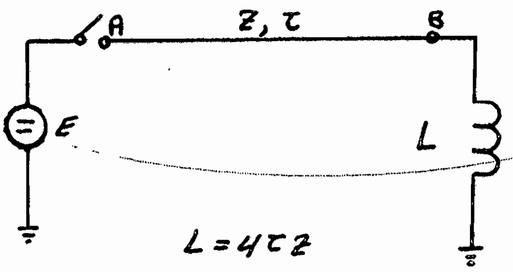
Fig. (19)



Voltage at A, B, and X

9. Line terminated with inductance

Energization is from dc source.



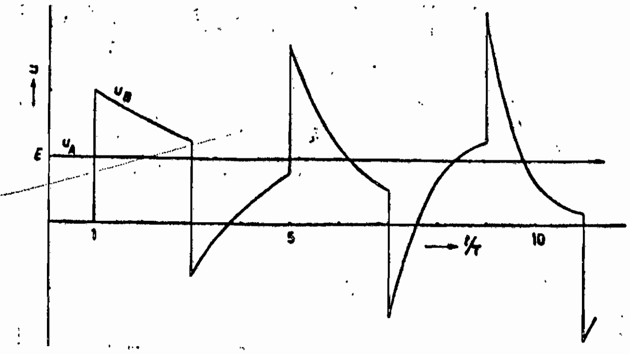
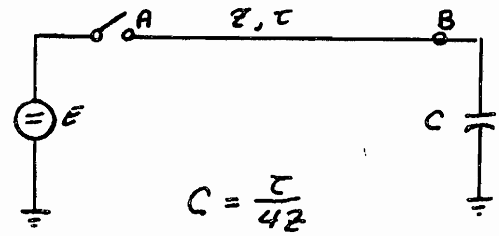


Fig. (20)

Voltage at A and B

10. Line terminated with capacitance

Energization is from dc source.



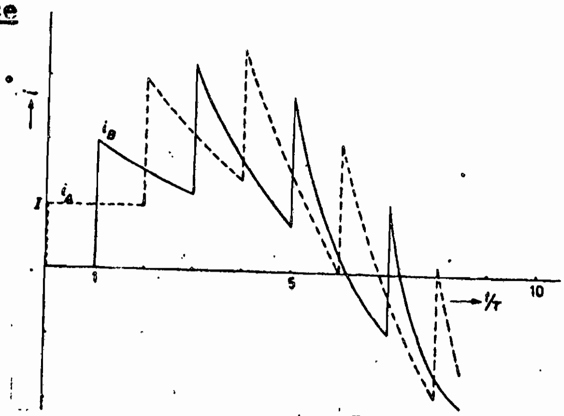
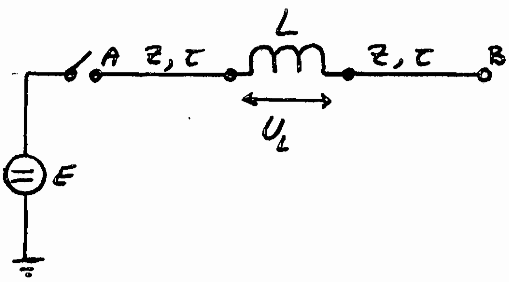


Fig. (21) In contrast to lumped inductances, sudden jumps in the current are possible in distributed-parameter lines

11. Lumped series inductance



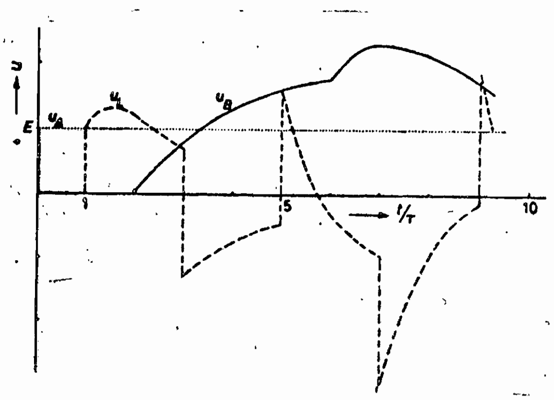


Fig. (22)

12. Lumped series capacitance

Energization is from dc source.

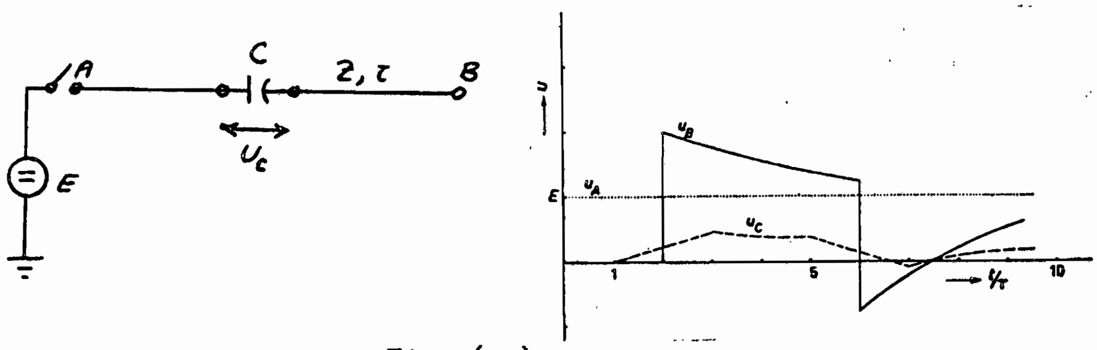


Fig. (23)

Voltage at A and B

13. Shunt inductance in middle of line

Energization is from dc source.

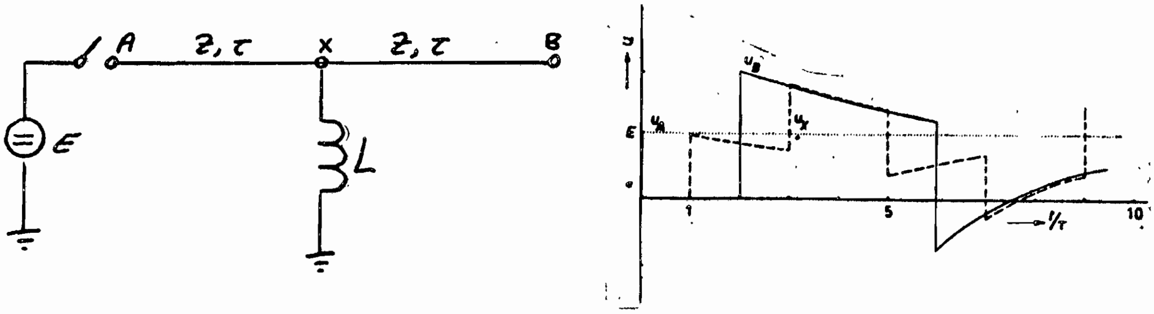


Fig. (24)

Voltage at A, B, and X

14. Shunt capacitance in middle of line

Energization is from dc source.

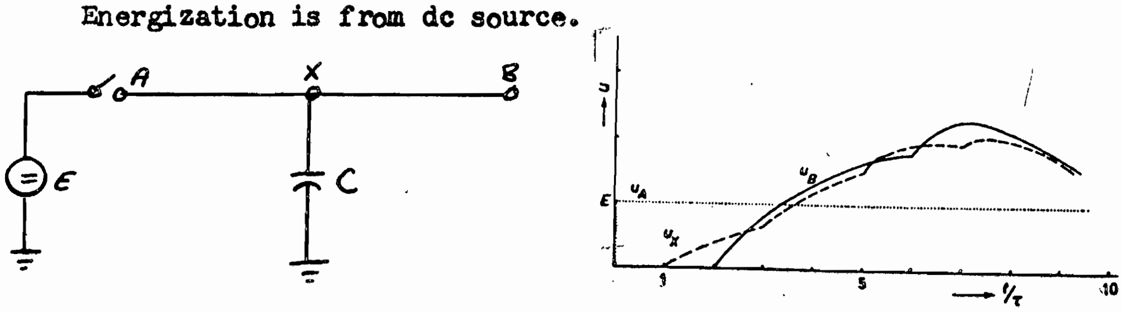


Fig. (25)

<u>Lumped parameters</u>

Lumped parameters are handled by the use of the trapezoidal rule of integration using digital computers. The impedance network is determined by the equivalent impedance of each element.

<u>Inductance</u>. Consider an inductance between nodes a and b in fig. (26). The voltage across the inductance is:

$$e_a(t) - e_b(t) = L(di_{a,b}(t)/dt) \qquad (46)$$

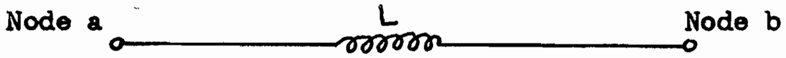


Fig. (26)

Let us assume that voltage and current at time $t - \Delta t$ are known, and we want to find voltage and current at time t. These are determined by integration of equation (46) from $t - \Delta t$ to the unknown state t. From equation (46) we have:

$$di_{a,b}(t) = (1/L) \ (e_a(t) - e_b(t))dt$$

$$\int_{t-\Delta t}^{t} di_{a,b}(t) = (1/L) \int_{t-\Delta t}^{t} (e_a(t) - e_b(t))dt$$

$$i_{a,b}(t) = (1/L) \int_{t-\Delta t}^{t} (e_a(t) - e_b(t))dt$$

$$i_{a,b}(t) = i_{a,b}(t - \Delta t) + (1/L) \int_{t-\Delta t}^{t} (e_a(t) - e_b(t))dt \quad (47)$$

The second term of right hand side of equation (47) can be integrated by using the trapezoidal rule of integration and this gives:

$$i_{a,b}(t) = i_{a,b}(t - \Delta t) + (1/L)(\Delta t/2)(e_a(t) - e_b(t) + e_a(t - \Delta t) - e_b(t - \Delta t))$$

Rearranging this equation, we have:

$$i_{a,b}(t) = (\Delta t/2L)(e_a(t) - e_b(t)) + i_{a,b}(t - \Delta t) + (\Delta t/2L)(e_a(t - \Delta t) - e_b(t - \Delta t)) \quad (48)$$

Let the second half right hand side of equation (48) be designated by $I_{a,b}(t - \Delta t)$, then equation (48) can be written in the form of:

$$i_{a,b}(t) = (\Delta t/2L)(e_a(t) - e_b(t)) + I_{a,b}(t - \Delta t) \quad (49)$$

where

$$I_{a,b}(t - \Delta t) = i_{a,b}(t - \ t) + (\Delta t/2L)(e_a(t - \Delta t) - e_b(t - \Delta t)). \quad (50)$$

Now by considering equations (49) and (50), $(\Delta t/2L)$ and $I_{a,b}(t - \Delta t)$ can be simulated in a resistance and source current across the resistor, respectively. Therefore the equivalent impedance network of an inductor can be illustrated as in fig. (27).

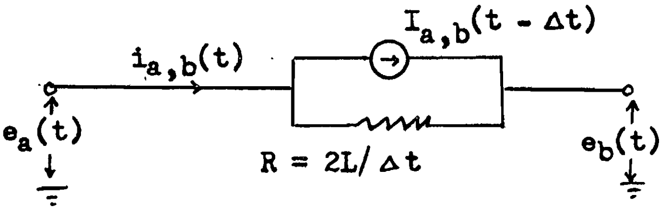


Fig. 27 Equivalent impedance network

As was mentioned before, the trapezoidal rule produces some error in computation and this error is of order $(\Delta t)^3$; now if Δt be chosen sufficiently small and cut in half, then the error will be cut in 1/8. The trapezoidal rule of integration used in equation (48) shows that this rule is identical with replacing the differential quotient in (46) by a central difference quotient at midpoint between t and $t - \Delta t$ with assuming that e is linear during this interval.

<u>Capacitance</u>

Consider a capacitor as shown in fig. (28). The voltage across a capacitor in general is:

$$e(t) = 1/C \int idt + e(0) \qquad (51)$$

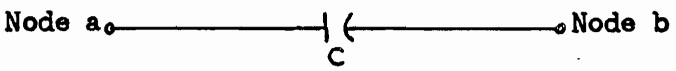


Fig. (28)

We wish to find the voltage across the capacitor at time (t), while we know voltage and current at time $(t - \Delta t)$. Therefore equation (51) can be written in the following form:

$$e_a(t) - e_b(t) = 1/C \;\; idt + e_a(t - \;\; t) - e_b(t - \;\; t). \qquad (52)$$

Let us apply the trapezoidal rule of integration on equation (52), which yields:

$$e_a(t) - e_b(t) - (1/c)(\Delta t/2)(i_{a,b}(t) + i_{a,b}(t - \Delta t)) + e_a(t - \Delta t) - e_b(t - \Delta t) \qquad (53)$$

Solving equation (53) for $i_{a,b}(t)$, it gives:

$$i_{a,b}(t) = (2C/\Delta t)(e_a(t) - e_b(t)) - i_{a,b}(t - \Delta t) - (2C/\Delta t)(e_a(t - \Delta t) - e_b(t - \Delta t)) \qquad (54)$$

If the second half right side of equation (54) is designated as $I_{a,b}(t - \Delta t)$, equation (54) can be written in the form of:

$$i_{a,b}(t) = (2C/\Delta t)(e_a(t) - e_b(t)) + I_{a,b}(t - \Delta t) \qquad (55)$$

where

$$I_{a,b}(t - \Delta t) = -i_{a,b}(t - \Delta t) - (2C/\Delta t)(e_a(t - \Delta t) - e_b(t - \Delta t)) \qquad (56)$$

Again by considering equations (55) and (56), $(2C/\Delta t)$ and

Now if these four nodes can be reduced to two nodes, it would be easier to solve a system than using 4 nodes. Suppose there is a ten-circuit series of R - L - C branches in a system, and if for each element we use one equivalent network impedance, we have to deal with 28 nodes. However, if each of the R - L - C branches can be presented by two nodes and one equivalent resistance, we have to deal with only 10 nodes. This will be illustrated in the next digital computer solution for a series R - L - C branch.

In order to derive an equation describing the relation of voltage and current in a series R - L - C branch, consider fig. (31)a. The fundamental equation is:

$$e_a - e_b = Ri + L(di/dt) + 1/C \int idt + e_c(0) \qquad (58)$$

where $e_c(0)$ is the initial condition of voltage e_c across the capacitor.

Let us apply the trapezoidal rule of integration to equation (58). We know this rule is equivalent to using central differences; this means $e = L(di/dt)$ can be replaced by:

$$(1/2)(e(t) + e(t - \Delta t)) = (L/\Delta t)(i(t) - i(t - \Delta t)) \quad (59)$$

This is similar to equation (49). Now the average capacitor voltage between $(t - \Delta t)$ and (t) is:

$$e_c(\text{average}) = (1/2)(e_c(t) + e_c(t - \Delta t))$$

$e_c = 1/C \int idt$ can be written in the form of $i = C(de_c/dt)$, by the same procedure used for equation (58), $i = c(de_c/dt)$ can be written as:

$$(1/2)(i(t) + i(t - \Delta t)) = (C/\Delta t)(e_c(t) - e_c(t - \Delta t))$$

or

$$e_c = (\Delta t/2c)(i(t) + i(t - \Delta t)) + e_c(t - \Delta t)$$

Now let us determine each term of equation (58) step by step at $(t - \Delta t)$ to (t), and then substitute each term in the equation and find the final result which is applicable for our digital computer solution.

$$e_a - e_b = (1/2)(e_a(t) - e_b(t) + e_a(t - \Delta t) - e_b(t - \Delta t))$$

$$Ri = (R/2)(i(t) + i(t - \Delta t))$$

$$L(di/dt) = (L/\Delta t)(i(t) - i(t - \Delta t))$$

$$(1/C)\int idt + e_c(0) = (1/2)(e_c(t) + e_c(t - \Delta t) = (\Delta t/4C)(i(t) + i(t - \Delta t)) + e_c(t - \Delta t)$$

Now substitute each term in equation (58) and multiply by two and reorder; we have:

$$(e_a(t) - e_b(t)) + (e_a(t - \Delta t) - e_b(t - \Delta t)) = i(t)(R + 2L/\Delta t + \Delta t/2C) + i(t - \Delta t)(R - 2L/\Delta t + \Delta t/2C) + 2e_c(t - \Delta t) \quad (59)$$

Let the following symbols be assigned:

$$Y = 1/(R + 2L/\Delta t + \Delta t/2C)$$

$$P = Y(R - 2L/\Delta t + \Delta t/2C)$$

Then equation (59) becomes:

$$i_{a,b}(t) = Y(e_a(t) - e_b(t)) + I_{a,b}(t - \Delta t) \quad (60)$$

where

$$I_{a,b}(t - \Delta t) = Y(e_a(t - \Delta t) - e_b(t - \Delta t) - 2e_c(t - \Delta t)) - Pi_{a,b}(t - \Delta t) \quad (61)$$

For computer programming the updating of $I_{a,b}$ can be done as follows:

a) Compute $H = Y(e_a(t) - e_b(t)) + I_{a,b}(t - \Delta t)$.

This is the new current, but it should not be stored.

b) Compute $e_c(t) = e_c(t - \Delta t) + (\Delta t/2C)(i(t - \Delta t) + H)$

c) Update $I_{a,b}(t) = Y(e_a(t) - e_b(t) - 2e_c(t)) - PH$

d) H should be stored into location for current, $i(t) = H$

For past history we should have three values I_a, b, i, and e_c.

By considering equations (60) and (61), the equivalent impedance network for a series R - L - C is as shown in fig. (32).

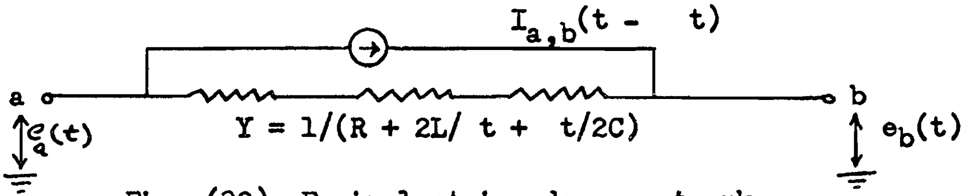


Fig. (32) Equivalent impedance network

When all impedance networks are replaced by each element, the nodal equations for any network can be determined. This was explained in previous sections. The set of equations can be written in the form of:

$$\left[Y\right] \cdot \left[e(t)\right] = \left[i(t)\right] - \left[I\right]$$

Matrix $\left[e(t)\right]$, $\left[i(t)\right]$, and $\left[I\right]$ are column matrix. Part of the voltage of $\left[e(t)\right]$ matrix is known and the other part is unknown. Let K designate the known part and U the unknown part; then we have:

$$\begin{bmatrix} [Y_{UU}] & [Y_{UK}] \\ [Y_{KU}] & [Y_{KK}] \end{bmatrix} \begin{bmatrix} [e_U(t)] \\ [e_K(t)] \end{bmatrix} = \begin{bmatrix} i_U(t) \\ i_K(t) \end{bmatrix} - \begin{bmatrix} I_U \\ I_K \end{bmatrix}$$

Solving for $\left[e_U(t)\right]$, it gives:

$$\left[Y_{UU}\right] \left[e_U(t)\right] = \left[I_T\right] - \left[Y_{UK}\right] \left[e_K(t)\right]$$

or

$$\left[e_U(t)\right] = \left[Y_{UU}\right] \left[I_T\right] - \left[Y_{UK}\right] \left[e_K(t)\right] \qquad (61)$$

where

$$\left[I_T\right] = \left[i_U(t)\right] - \left[I_U\right]$$

This leads us to the solution of a system of linear equations with a constant coefficient matrix $\left[Y_{UU}\right]$, and with Δt fixed.

A digital computer solution is illustrated in full details on the next page.

Test Case No. 2

An impulse of rectangular waveform is applied to a voltage divider. This voltage divider has an equivalent circuit as shown in fig. (33). The equivalent circuit is formed of ten equal sections. We wish to determine the oscillation in voltage at low voltage end. Voltage at node 10 and current from node 9 to 10 are desired.*

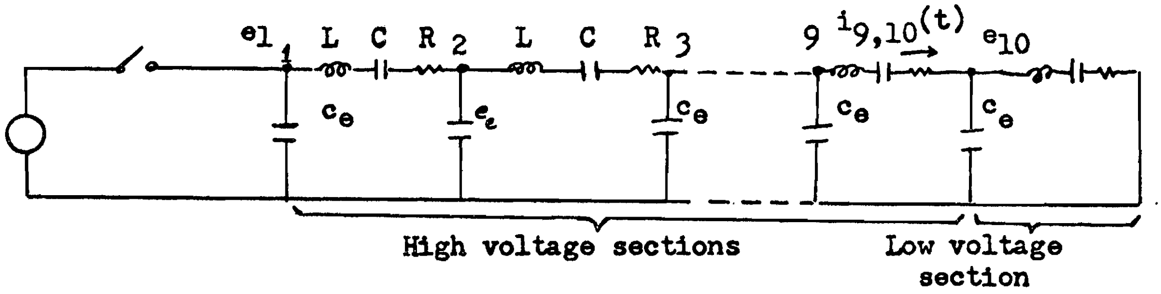


Fig. (33)

Given:

$C_e = 10$ PF

$L = .5\ \mu H$

$C = 15$ nF

$R = 1\Omega$

$\Delta t = 1$ ns

Excitation:

$e_1(t) = 1.$ for $t > 0$

$e_1(t) = 0$ for $t \leq 0$

Solution:

By considering fig. (32) for a series R - L - C and fig. (27) for a capacitor, the equivalent impedance network for the system shown in fig. (33) can be illustrated as in fig. (34) on the next page.

Let the following symbols be assigned:

$$Y = 1/(R + 2L/\Delta t + \Delta t/2C)$$

$$P = Y(R - 2L/\ t + \Delta t/2C)$$

*This problem is taken from Hermann W. Dammel, Habilitation Thesis, submitted to the Munich Institute of Technology, May 1967, p.37.

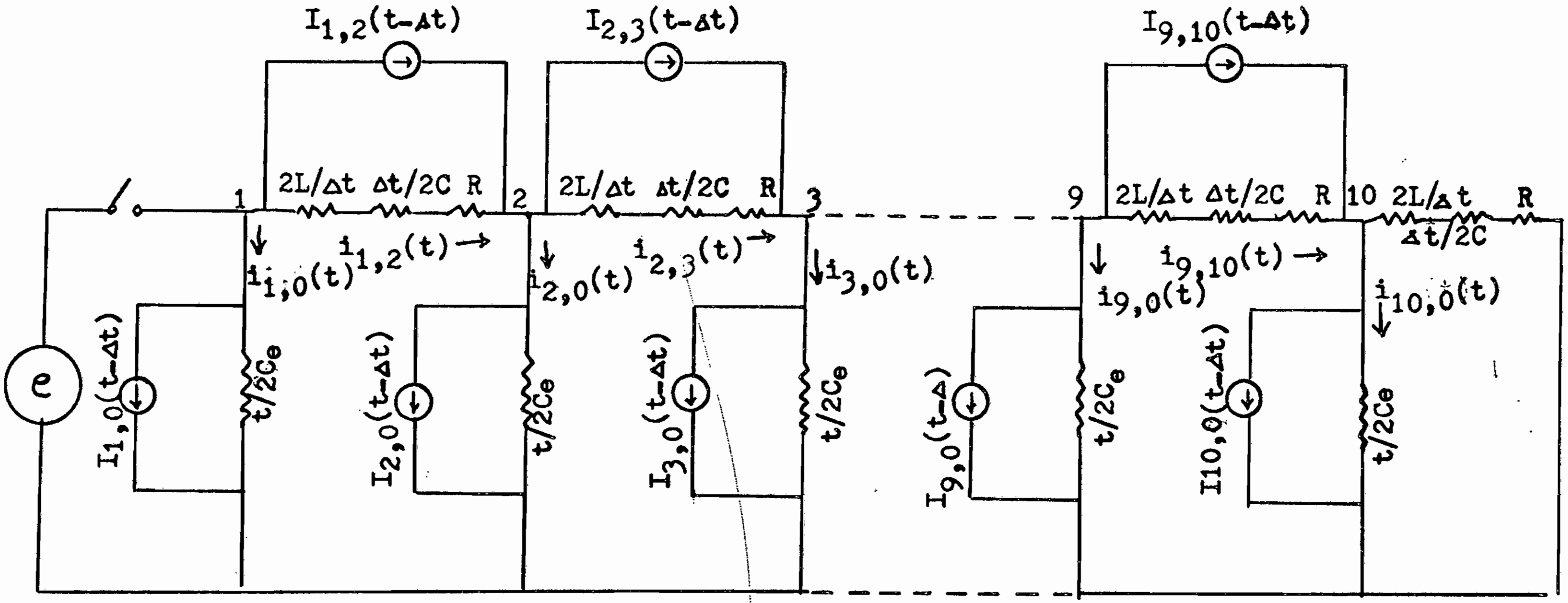


Fig. (34) Equivalent Impedance Network

Node equations:

Since we know voltage at node 1, there is no need to write the node equation for this node.

Node No. 2:

$$-i_{1,2} + i_{2,0} + i_{2,3} = 0$$

according to equations (55) and (60), we have:

$$-i_{1,2}(t) = -Y(e_1(t) + e_2(t)) - I_{1,2}(t - \Delta t)$$

$$i_{2,0}(t) = 2C_e/\Delta t(e_2(t) - 0) + I_{2,0}(t - \Delta t)$$

$$i_{2,3}(t) = Y(e_2(t) - e_3(t)) + I_{2,3}(t - \Delta t)$$

By adding and reordering these equations, it gives:

$$-Ye_1(t) + (2Y + 2C_e/\Delta t)e_2(t) - Ye_3(t) - I_{1,2}(t - \Delta t) + I_{2,0}(t - \Delta t) + I_{2,3}(t - \Delta t)$$

let:

$$Y' = 2Y + 2C_e/\Delta t$$

$$I_1 = Ye_1(t)$$

$$I_{T2} = -I_{1,2}(t - \Delta t) + I_{2,0}(t - \Delta t) + I_{2,3}(t - \Delta t)$$

then the node equation for node No. 2 is:

$$I_1 = Y'e_2(t) - Ye_3(t) + I_{T2}$$

Node No. 3:

$$-i_{2,3}(t) + i_{3,0}(t) + i_{3,4}(t) = 0$$

where

$$-i_{2,3}(t) = -Y(e_2(t) - e_3(t)) - I_{2,3}(t - \Delta t)$$

$$i_{3,0}(t) = 2C_e/\Delta t(e_3(t) - 0) + I_{3,0}(t - \Delta t)$$

$$i_{3,4}(t) = Y(e_3(t) - e_4(t)) + I_{3,4}(t - \Delta t).$$

Again, by adding and reordering these equations and letting

$I_{T3} = -I_{2,3}(t - \Delta t) + I_{3,0}(t - \Delta t) + I_{3,4}(t - \Delta t)$, it gives:

$$0 = -Ye_2(t) + Y' e_3(t) - Ye_4(t) + I_{T3}$$

Similarly, for nodes 4, 5, ... and 10, we have:

$$0 = -Ye_3(t) + Y'e_4(t) - Ye_5(t) + I_{T4}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots$$

$$\cdots\cdots\cdots\cdots\cdots\cdots$$

$$0 = -Ye_8(t) + Y'e_9(t) - Ye_{10}(t) + I_{T9}$$

$$0 = -Ye_9(t) + Y'e_{10}(t) + I_{T10}$$

In matrix form this can be written as follows:

$$\begin{bmatrix} I_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} Y' & -Y & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -Y & Y'-Y & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -Y & Y'-Y & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -Y & Y'-Y & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -Y & Y' & -Y & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -Y & Y'-Y & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -Y & Y'-Y & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -Y & Y'-Y & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -Y & Y \end{bmatrix} \begin{bmatrix} e_2 \\ e_3 \\ e_4 \\ e_5 \\ e_6 \\ e_7 \\ e_8 \\ e_9 \\ e_{10} \end{bmatrix} + \begin{bmatrix} I_{T2} \\ I_3 \\ I_{T4} \\ I_{T5} \\ I_{T6} \\ I_{T7} \\ I_{T8} \\ I_{T9} \\ I_{T10} \end{bmatrix}$$

which is:

$$[i] = [Y][e] + [I_T]$$

$$[e] = [Y]^{-1}([i] - [I_T])$$

Now we are ready to write a computer program for this system. Let us first draw a flow chart for the program and then write the computer program.

In order to be able to follow the program, let us define the symbols used in the program.

T = maximum time

D = Δt

R1 = R

L2 = L

C = series capacitance

C1 = C_e = shunt capacitance

$[A] = [I_{n-1,\,n}]$

$[B] = [I_{n,0}]$ $\quad n = 1,2,3, \ldots\ldots, 9$

$[C] = [I_{n,\,n+1}]$

$[A] - [B] - [C] = [I_T]$

$[Z] = [i]$

$[Q] = [Y]$

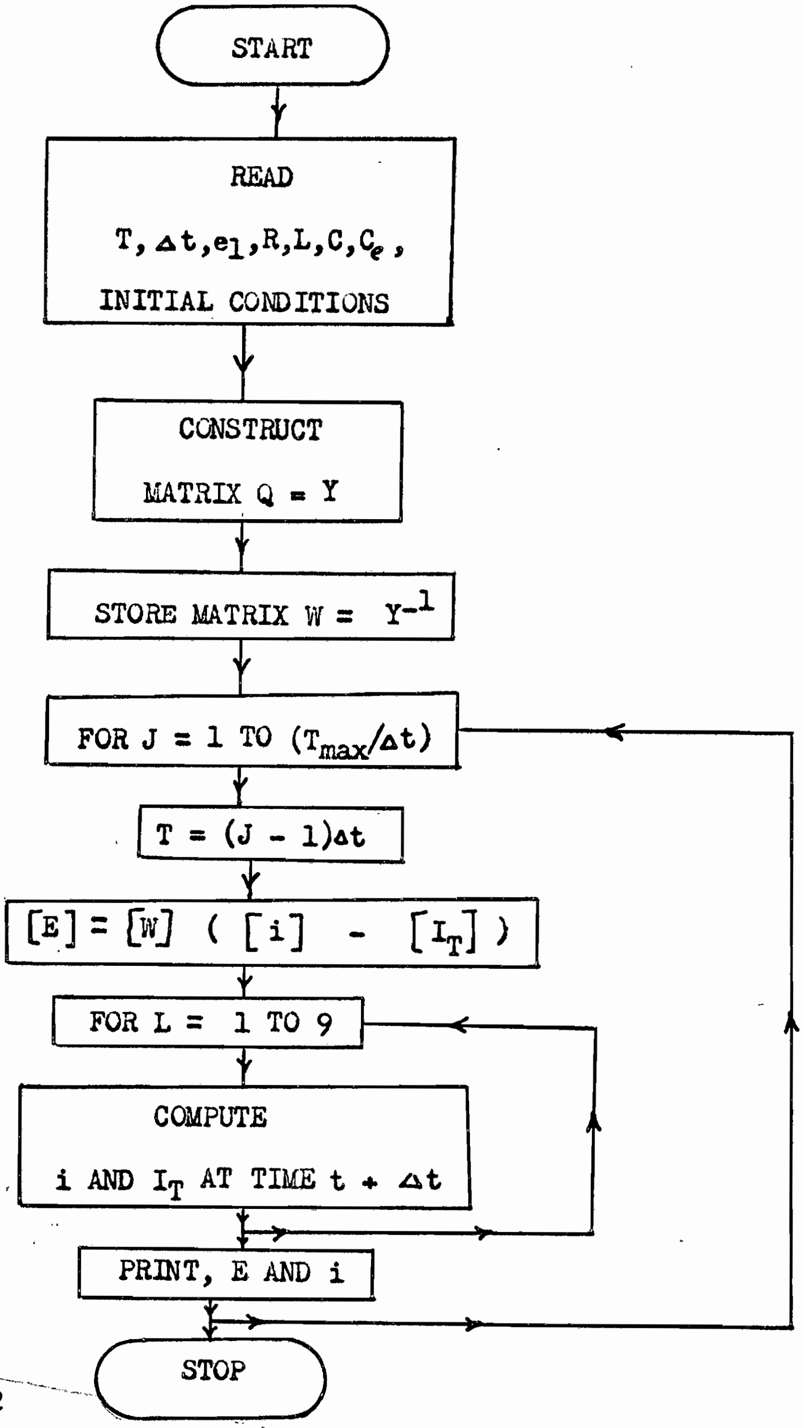
$[W] = [Y]^{-1}$

J1 = maximum time

R(L,M) = $i_{L,L+1}$ $\quad L = 1,2,3,\ldots,9$

M = 1

The flow chart is shown on the next page.



Flow chart of
test case No. 2

Computer program:

```
100 DIM E(9,1),Z(9,1),A(9,1),B(9,1),C(9,1)
110 DIM V(9,1),R(9,1),I(9,1),T(9,1),P(9,1)
120 DIM W(9,9),F(9,1),G(9,1),H(9,1),Q(9,9)
130 READ T,D,E1
140 READ R1,L2,C,C1,A1,V1,I1
150 MAT A=ZER(9,1)
160 MAT B=ZER(9,1)
170 MAT C=ZER(9,1)
180 MAT Z=ZER(9,1)
190 MAT I=ZER(9,1)
200 MAT V=ZER(9,1)
210 MAT P=ZER(9,1)
220 PRINT "TIME","CURRENT TO 10","VOLTAGE AT 10"
225 PRINT
230 Y=1/(R1+2*L2/D+D/(2*C))
240 U=Y*(R1-2*L2/D+D/(2*C))
250 Y1=2*Y+2*C1/D
260 MAT Q=ZER(9.9)
270 FOR K2=1 TO 9
280 FOR K3=1 TO 9
290 IF K2<>K3 THEN 370
300 Q(K2,K3)=Y1
310 K4=K3+1
320 IF K4=10 THEN 380
330 Q(K2,K4)=-Y
```

```
340 K5=K3-1
350 IF K5=0 THEN 370
360 Q(K2,K5)=-Y
370 NEXT K3
380 NEXT K2
390 MAT W=INV(Q)
400 J1=1+T/D
410 FOR J=1 TO J1
420 MAT F=A-B
430 MAT G=F-C
440 MAT H=G+Z
450 MAT E=W*H
460 FOR L=1 TO 9
470 M=1
480 IF L>1 THEN 550
490 IF J= THEN 520
500 S=E1
510 GO TO 530
520 S=0.
530 R(L,M)=Y*(S-E(1,1))+A(1,1)
540 GO TO 570
550 L1=l-1
560 R(L,M)=Y*(E(L1,M)-E(L,M))+A(L,M)
570 V(L,M)=V(L,M)+(D/(2*C))*(I(L,M)+R(L,M))
580 IF L>1 THEN 650
590 IF J=1 THEN 620
```

```
600 S=E1
710 GO TO 630
610 S=0.
630 T(L,M)=Y*(S-E(L,M)-2*V(L,M))-U*R(L,M)
640 GO TO 660
650 T(L,M)=Y*(E(L1,M)-E(L,M)-2*V(L,M))-U*R(L,M)
660 I(L,M)=R(L,M)
670 A(L,M)=T(L,M)
680 P(L,M)=(2*C1/D)*E(L,M)+B(L,M)
690 B(L,M)=-P(L,M) - (2*C1/D)*E(L,M)
700 NEXT L
710 FOR N=1 to 9
720 N1=N+1
730 IF N1=10 THEN 760
740 C(N,M)=A(N1,M)
750 GO TO 810
760 R2=Y*E(9,1)+A1
770 V1=V1+(D/(2*C))*(I1+R2)
780 A1=Y*(E(9,M)-2*V1)-U*R2
790 I1=R2
800 C(9,M)=A1
810 NEXT N
820 Z(1,1)=Y*E1
830 K1=J-1
840 T1=K1*D
850 PRINT T1,I(9,1),E(9,1)
```

```
860 NEXT J
870 DATA 1E-7,2E-9,1.
880 DATA 1,.5E-6,15E-9,1E-11,.0,.0,.0
999 END
```

This program was executed by G.E. MARK II in basic language. The result is shown on the next page.

Δt = 1 nSec.

TIME	CURRENT TO 10	VOLTAGE AT 10
0	0	0
1.E-9	1.83314E-14	0
2.E-9	5.88344E-13	1.66487E-12
3.E-9	9.35622E-12	5.46451E-11
4.E-9	9.83733E-11	8.89194E-10
5.E-9	7.69899E-10	9.57107E-9
6.E-9	4.78733E-9	7.6715E-8
7.E-9	2.46517E-8	4.88698E-7
8.E-9	1.08181E-7	2.57863E-6
9.E-9	4.13185E-7	1.15968E-5
1.E-8	1.39571E-6	4.53916E-5
1.1E-8	4.22254E-6	1.57112E-4
1.2E-8	1.15576E-5	4.86907E-4
1.3E-8	2.88562E-5	1.36457E-3
1.4E-8	6.61632E-5	3.48603E-3
1.5E-8	1.40091E-4	8.17068E-3
1.6E-8	2.75178E-4	1.76621E-2
1.7E-8	5.03352E-4	3.53576E-2
1.8E-8	8.60069E-4	0.065759
1.9E-8	1.37627E-3	0.113879
2.E-8	2.06678E-3	0.183878
2.1E-8	2.91792E-3	0.276932
2.2E-8	3.87927E-3	0.388708
2.3E-8	4.86513E-3	0.507324
2.4E-8	5.76935E-3	0.613001
2.5E-8	6.4928E-3	0.680521
2.6E-8	0.006976	0.684931
2.7E-8	7.22497E-3	0.609648
2.8E-8	7.31685E-3	0.454632
2.9E-8	7.37774E-3	0.241327
3.E-8	7.53591E-3	1.12219E-2
3.1E-8	7.86628E-3	-0.183325
3.2E-8	8.34976E-3	-0.29445
3.3E-8	8.86951E-3	-0.295591
3.4E-8	9.25222E-3	-0.193238
3.5E-8	9.34264E-3	-2.78355E-2
3.6E-8	9.08076E-3	0.138319
3.7E-8	8.54481E-3	0.243057
3.8E-8	7.93436E-3	0.248175
3.9E-8	7.49371E-3	0.155202
4.E-8	7.40553E-3	5.53983E-3
4.1E-8	7.70319E-3	-0.136309
4.2E-8	8.24641E-3	-0.210143
4.3E-8	8.77769E-3	-0.186373
4.4E-8	9.03962E-3	-7.92247E-2
4.5E-8	8.89569E-3	5.97903E-2
4.6E-8	8.40122E-3	0.165742

TIME	CUURENR AT 10	VOLTAGE AT 10
4.7E-8	7.78015E-3	0.190656
4.8E-8	7.31874E-3	0.126015
4.9E-8	7.22514E-3	6.35218E-3
5.E-8	7.5251E-3	-0.108057
5.1E-8	8.04834E-3	-0.161066
5.2E-8	8.51476E-3	-0.128291
5.3E-8	8.67823E-3	-2.86038E-2
5.4E-8	8.4541E-3	8.63374E-2
5.5E-8	7.96359E-3	0.159137
5.6E-8	7.47106E-3	0.156318
5.7E-8	7.24635E-3	8.52172E-2
5.8E-8	7.42512E-3	-0.010585
5.9E-8	7.94169E-3	-7.46785E-2
6.E-8	8.5702E-3	-6.75357E-2
6.1E-8	9.05161E-3	1.33047E-2
6.2E-8	9.23714E-3	0.133167
6.3E-8	9.16982E-3	0.239952
6.4E-8	9.05985E-3	0.292618
6.5E-8	9.16773E-3	0.282812
6.6E-8	9.66017E-3	0.237974
6.7E-8	1.05179E-2	0.204554
6.8E-8	1.15458E-2	0.220957
6.9E-8	1.24773E-2	0.295312
7.E-8	1.31141E-2	0.400287
7.1E-8	1.34229E-2	0.487499
7.2E-8	1.35359E-2	0.513191
7.3E-8	1.36594E-2	0.460817
7.4E-8	1.39441E-2	0.348259
7.5E-8	1.43938E-2	0.216385
7.6E-8	1.48648E-2	0.106184
7.7E-8	1.51538E-2	3.78788E-2
7.8E-8	1.51238E-2	3.7615E-3
7.9E-8	1.47912E-2	-2.18747E-2
8.E-8	1.43259E-2	-6.24451E-2
8.1E-8	0.013961	-0.121122
8.2E-8	1.38659E-2	-0.176343
8.3E-8	1.40537E-2	-0.193775
8.4E-8	1.43762E-2	-0.147893
8.5E-8	1.46078E-2	-4.04611E-2
8.6E-8	1.45696E-2	9.52414E-2
8.7E-8	1.42261E-2	0.207631
8.8E-8	1.36995E-2	0.250847
8.9E-8	0.0132	0.206911
9.E-8	1.29124E-2	9.44087E-2
9.1E-8	1.29046E-2	-4.05258E-2
9.2E-8	1.31053E-2	-0.145744
9.3E-8	1.33563E-2	-0.18577
9.4E-8	1.35048E-2	-0.154581
9.5E-8	1.34784E-2	-7.35513E-2

TIME	CURRENT AT 10	VOLTAGE AT 10
9.6E-8	1.33051E-2	0.02227
9.7E-8	1.30739E-2	0.100891
9.8E-8	1.28697E-2	0.144475
9.9E-8	0.012726	0.150904
1.E-7	1.26239E-2	0.128056
1.01E-7	1.25315E-2	8.66966E-2
1.02E-7	1.24508E-2	3.70862E-2
1.03E-7	1.24332E-2	-9.79344E-3
1.04E-7	0.01255	-3.97979E-2
1.05E-7	1.28348E-2	-3.66782E-2
1.06E-7	1.32396E-2	0.01137
1.07E-7	1.36394E-2	0.102543
1.08E-7	1.38931E-2	0.216033
1.09E-7	1.39293E-2	0.316089
1.1E-7	1.38052E-2	0.366209
1.11E-7	0.013697	0.347509
1.12E-7	1.38175E-2	0.271337
1.13E-7	1.43008E-2	0.177788
1.14E-7	1.51165E-2	0.118627
1.15E-7	1.60649E-2	0.131876
1.16E-7	1.68632E-2	0.220799
1.17E-7	1.72814E-2	0.348943
1.18E-7	1.72572E-2	0.455176
1.19E-7	1.69289E-2	0.482297
1.2E-7	1.65652E-2	0.405269
1.21E-7	1.64275E-2	0.244671
1.22E-7	1.66355E-2	5.81226E-2
1.23E-7	1.71048E-2	-8.64731E-2
1.24E-7	1.75908E-2	-0.142712
1.25E-7	1.78137E-2	-0.106737
1.26E-7	1.76019E-2	-1.72271E-2
1.27E-7	1.69808E-2	6.53583E-2
1.28E-7	1.61623E-2	8.96239E-2
1.29E-7	1.54441E-2	3.87242E-2
1.3E-7	1.50683E-2	-6.19654E-2
1.31E-7	1.51108E-2	-0.15835
1.32E-7	1.54514E-2	-0.195195
1.33E-7	1.58377E-2	-0.144039
1.34E-7	1.60068E-2	-1.79388E-2
1.35E-7	1.58069E-2	0.133763
1.36E-7	1.52621E-2	0.247805
1.37E-7	1.45536E-2	0.275743
1.38E-7	1.39315E-2	0.20569
1.39E-7	1.35983E-2	6.73467E-2
1.4E-7	0.01362	-8.14975E-2
1.41E-7	1.39018E-2	-0.180453
1.42E-7	1.42378E-2	-0.192338
1.43E-7	1.44095E-2	-0.117904

TIME	CURRENT AT 10	VOLTAGE AT 10
1.43E-7	1.44095E-2	-0.117904
1.44E-7	1.42875E-2	6.6233E-3
1.45E-7	1.38902E-2	0.127473
1.46E-7	1.33735E-2	0.198151
1.47E-7	1.29549E-2	0.199689
1.48E-7	1.28087E-2	0.147185
1.49E-7	1.29811E-2	8.00145E-2
1.5E-7	1.33671E-2	0.040813
1.51E-7	1.37635E-2	5.37142E-2
1.52E-7	1.39733E-2	0.11272
1.53E-7	0.013911	0.18611
1.54E-7	0.013654	0.234692
1.55E-7	0.013409	0.234392
1.56E-7	1.34058E-2	0.190996
1.57E-7	1.37706E-2	0.138472
1.58E-7	1.44448E-2	0.120792
1.59E-7	1.51989E-2	0.166278
1.6E-7	1.57401E-2	0.268349
1.61E-7	1.58639E-2	0.384181
1.62E-7	1.55689E-2	0.45379
1.63E-7	1.50707E-2	0.430961
1.64E-7	1.46984E-2	0.310075
1.65E-7	1.47224E-2	0.133728
1.66E-7	1.52015E-2	-2.47608E-2
1.67E-7	1.59348E-2	-9.66884E-2
1.68E-7	1.65495E-2	-5.36359E-2
1.69E-7	1.66874E-2	7.54772E-2
1.7E-7	1.61936E-2	0.2162
1.71E-7	1.52072E-2	0.284698
1.72E-7	1.41027E-2	0.230568
1.73E-7	1.33061E-2	6.42826E-2
1.74E-7	1.30815E-2	-0.145583
1.75E-7	1.33978E-2	-0.304853
1.76E-7	1.39491E-2	-0.339969
1.77E-7	1.43156E-2	-0.23448
1.78E-7	1.41831E-2	-3.71061E-2
1.79E-7	0.013503	0.162651
1.8E-7	1.25109E-2	0.278646
1.81E-7	1.15956E-2	0.269224
1.82E-7	1.10923E-2	0.154648
1.83E-7	1.11125E-2	2.90124E-3
1.84E-7	1.14975E-2	-0.107388
1.85E-7	1.19177E-2	-0.126711
1.86E-7	0.012055	-0.056575
1.87E-7	1.17683E-2	-5.65087E-2
1.88E-7	1.11523E-2	0.149643
1.89E-7	1.04668E-2	0.177916
1.9E-7	9.98251E-3	0.135898
1.91E-7	9.83641E-3	0.056009

TIME	CURRENT AT 10	VOLTAGE AT 10
1.92E-7	9.97531E-3	-1.29586E-2
1.93E-7	1.02118E-2	-3.34913E-2
1.94E-7	1.03503E-2	2.67175E-3
1.95E-7	1.03026E-2	0.074962
1.96E-7	1.01256E-2	0.150127
1.97E-7	9.9635E-3	0.20259
1.98E-7	9.93917E-3	0.226348
1.99E-7	1.00677E-2	0.232335
2.E-7	1.02488E-2	0.235097
2.01E-7	1.03428E-2	0.239353
2.02E-7	1.02774E-2	0.23642
2.03E-7	1.01135E-2	0.213164
2.04E-7	1.00194E-2	0.166859
2.05E-7	1.01638E-2	0.114489
2.06E-7	1.05921E-2	8.79079E-2
2.07E-7	1.11669E-2	0.115498
2.08E-7	1.16195E-2	0.200722
2.09E-7	0.011694	0.311877
2.1E-7	1.13024E-2	0.392215
2.11E-7	1.05976E-2	0.38779
2.12E-7	9.91028E-3	0.278866
2.13E-7	9.57517E-3	9.65193E-2
2.14E-7	9.73901E-3	-8.72938E-2
2.15E-7	1.02614E-2	-0.193623
2.16E-7	1.07728E-2	-0.177587
2.17E-7	1.08678E-2	-5.42053E-2
2.18E-7	1.03311E-2	0.106545
2.19E-7	9.26931E-3	0.214718
2.2E-7	8.07045E-3	0.207844
2.21E-7	7.20369E-3	8.52919E-2
2.22E-7	6.96648E-3	-8.98885E-2
2.23E-7	7.31796E-3	-0.223844
2.24E-7	7.89525E-3	-0.242508
2.25E-7	8.20916E-3	-0.132497
2.26E-7	7.91513E-3	5.05415E-2
2.27E-7	7.00975E-3	0.211483
2.28E-7	5.84081E-3	0.267608
2.29E-7	4.92032E-3	0.194388
2.3E-7	4.64045E-3	3.95428E-2
2.31E-7	5.04901E-3	-0.103124
2.32E-7	5.81197E-3	-0.146251
2.33E-7	6.38899E-3	-0.058886
2.34E-7	6.32962E-3	0.114315
2.35E-7	5.52831E-3	0.276272
2.36E-7	4.29533E-3	0.331236
2.37E-7	3.19973E-3	0.239825
2.38E-7	2.7651E-3	4.45249E-2

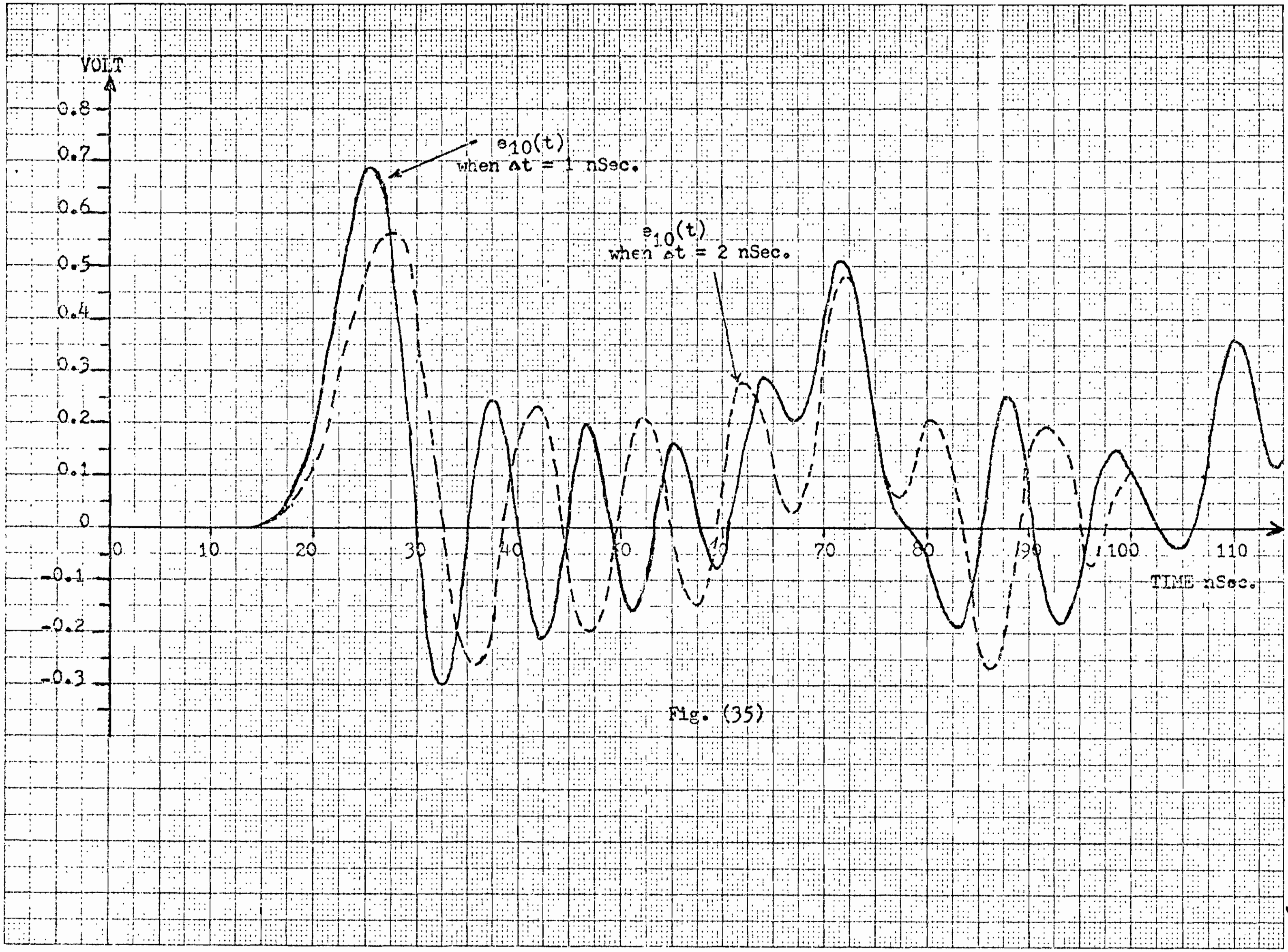
TIME	CURRENT AT 10	VOLTAGE AT 10
2.39E-7	3.18217E-3	-0.149642
2.4E-7	4.19587E-3	-0.230145
2.41E-7	5.23213E-3	-0.138557
2.42E-7	5.69991E-3	9.48079E-2
2.43E-7	5.30791E-3	0.362982
2.44E-7	4.22535E-3	0.536008
2.45E-7	2.99889E-3	0.528482
2.46E-7	2.26741E-3	0.345266
2.47E-7	2.42538E-3	8.07246E-2
2.48E-7	3.4128E-3	-0.128808
2.49E-7	4.74278E-3	-0.17491
2.5E-7	5.75176E-3	-3.29946E-2
2.51E-7	5.94051E-3	0.224853
2.52E-7	5.2274E-3	0.466737
2.53E-7	3.98376E-3	0.568315
2.54E-7	2.83635E-3	0.475838
2.55E-7	2.34293E-3	0.232651
2.56E-7	2.71001E-3	-4.38258E-2
2.57E-7	3.69574E-3	-0.222117
2.58E-7	4.7419E-3	-0.222588
2.59E-7	5.26087E-3	-5.59017E-2
2.6E-7	4.92751E-3	0.184502
2.61E-7	3.8296E-3	0.370802
2.62E-7	2.40819E-3	0.404667
2.63E-7	1.22764E-3	0.264509
2.64E-7	6.97811E-4	1.35849E-2
2.65E-7	8.87671E-4	-0.233296
2.66E-7	1.51552E-3	-0.366309
2.67E-7	2.10889E-3	-0.331969
2.68E-7	2.24461E-3	-0.155012
2.69E-7	1.74704E-3	7.77309E-2
2.7E-7	7.52774E-4	0.261284
2.71E-7	-3.78213E-4	0.32162
2.72E-7	-1.24915E-3	0.246578
2.73E-7	-1.6135E-3	8.57488E-2
2.74E-7	-1.47411E-3	-7.78279E-2
2.75E-7	-1.05636E-3	-0.16847

Δt = 2 nSec.

TIME	CURRENT TO 10	VOLTAGE AT 10
0	0	0
2.E-9	3.95973E-10	0
4.E-9	1.0477E-8	5.6474E-8
6.E-9	1.34325E-7	1.5024E-6
8.E-9	1.11072E-6	1.93426E-5
1.E-8	6.65004E-6	1.60322E-4
1.2E-8	3.0673E-5	9.59736E-4
1.4E-8	1.13195E-4	4.41043E-3
1.6E-8	3.42545E-4	1.61338E-2
1.8E-8	8.64058E-4	4.80359E-2
2.E-8	1.83661E-3	0.117875
2.2E-8	3.31295E-3	0.239446
2.4E-8	5.09707E-3	0.400804
2.6E-8	6.72786E-3	0.543081
2.8E-8	7.71656E-3	0.569764
3.E-8	7.93802E-3	0.408546
3.2E-8	7.80699E-3	9.72993E-2
3.4E-8	7.93606E-3	-0.192367
3.6E-8	8.49512E-3	-0.263324
3.8E-8	8.97341E-3	-7.58422E-2
4.E-8	8.75759E-3	0.177954
4.2E-8	7.96458E-3	0.235732
4.4E-8	7.45813E-3	3.90044E-2
4.6E-8	7.84149E-3	-0.18547
4.8E-8	8.61651E-3	-0.174086
5.E-8	8.72201E-3	5.55022E-2
5.2E-8	7.92955E-3	0.214789
5.4E-8	7.25074E-3	0.105492
5.6E-8	7.64892E-3	-0.116265
5.8E-8	8.70969E-3	-0.137492
6.E-8	9.18604E-3	9.15948E-2
6.2E-8	8.78844E-3	0.285833
6.4E-8	8.68646E-3	0.220399
6.6E-8	9.8775E-3	4.99707E-2
6.8E-8	1.16978E-2	8.72328E-2
7.E-8	1.37014E-2	0.34225
7.2E-8	1.27081E-2	0.491865
7.4E-8	1.29861E-2	0.340043
7.6E-8	1.42314E-2	9.99078E-2
7.8E-8	1.53735E-2	7.32841E-2
8.E-8	1.51042E-2	0.201004
8.2E-8	0.013891	0.166064

8.4E-8	1.33807E-2	-0.098411
8.6E-8	1.40175E-2	-0.274474
8.8E-8	1.44815E-2	-0.122471
9.E-8	1.38022E-2	0.153095
9.2E-8	1.28602E-2	0.197307
9.4E-8	1.29415E-2	1.45941E-2
9.6E-8	1.36543E-2	-7.88608E-2
9.8E-8	1.35628E-2	3.58113E-2
1.E-7	1.25707E-2	0.119863

The curves are shown on the following page.


VOLT
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
0
10
20
30
40
50
60
70
80
90
100
110
TIME nSec.
$e_{10}(t)$ when Δt = 1 nSec.
$e_{10}(t)$ when Δt = 2 nSec.
Fig. (35)

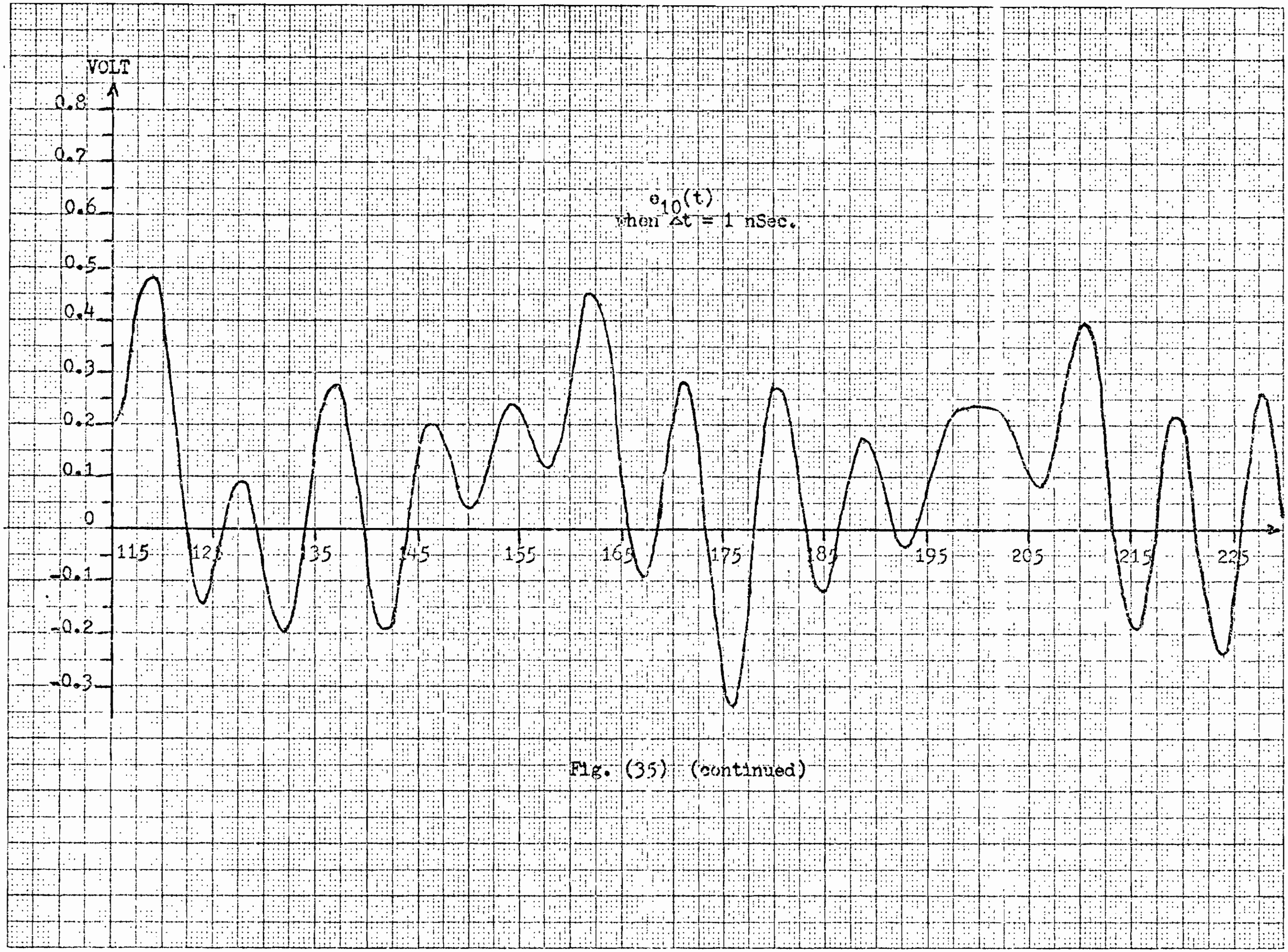


Fig. (35) (continued)

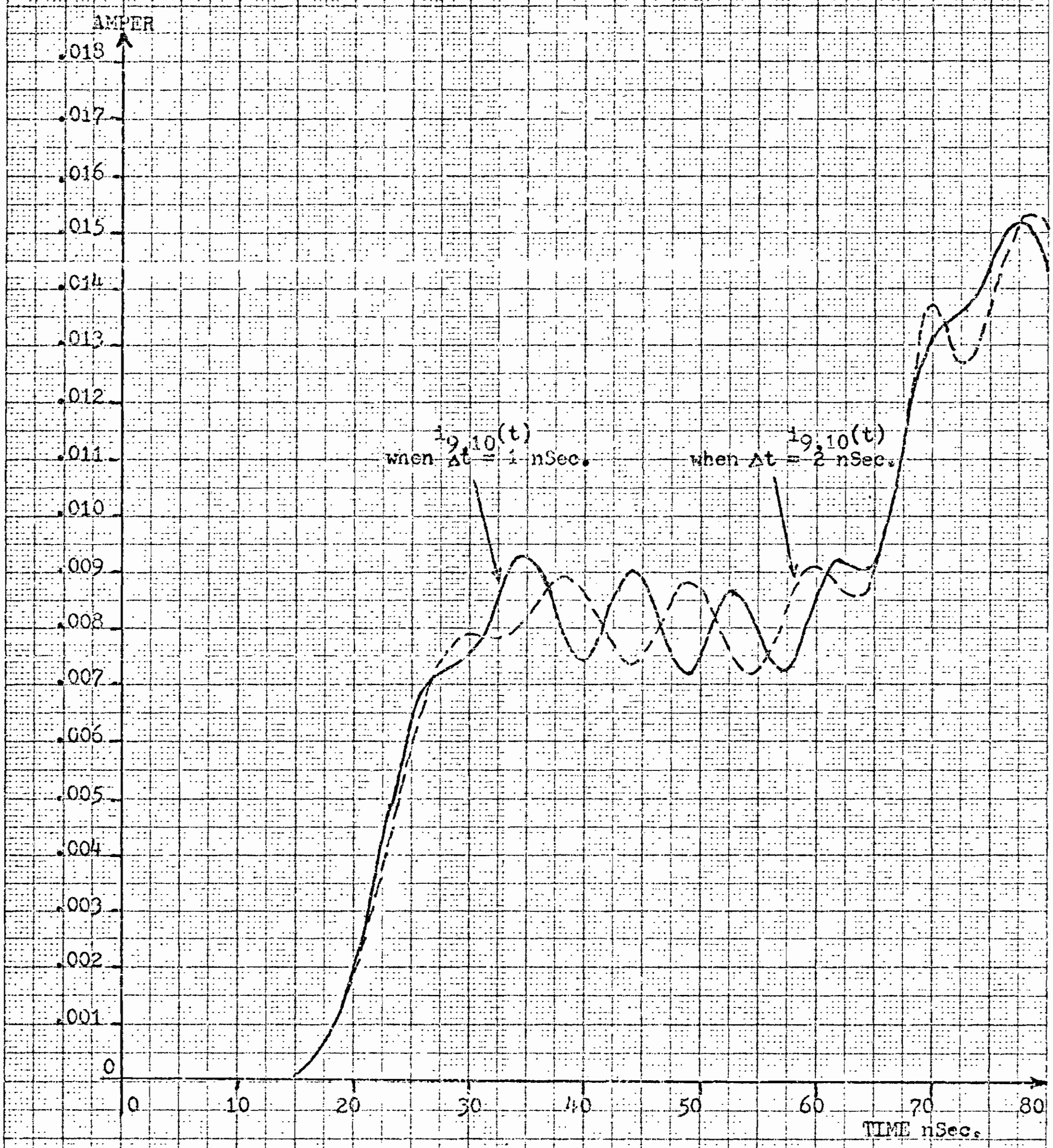

AMPER
.018
.017
.016
.015
.014
.013
.012
.011
.010
.009
.008
.007
.006
.005
.004
.003
.002
.001
0
i9,10(t)
when Δt = 1 nSec.
i9,10(t)
when Δt = 2 nSec.
0
10
20
30
40
50
60
70
80
TIME nSec.

Fig. (36)

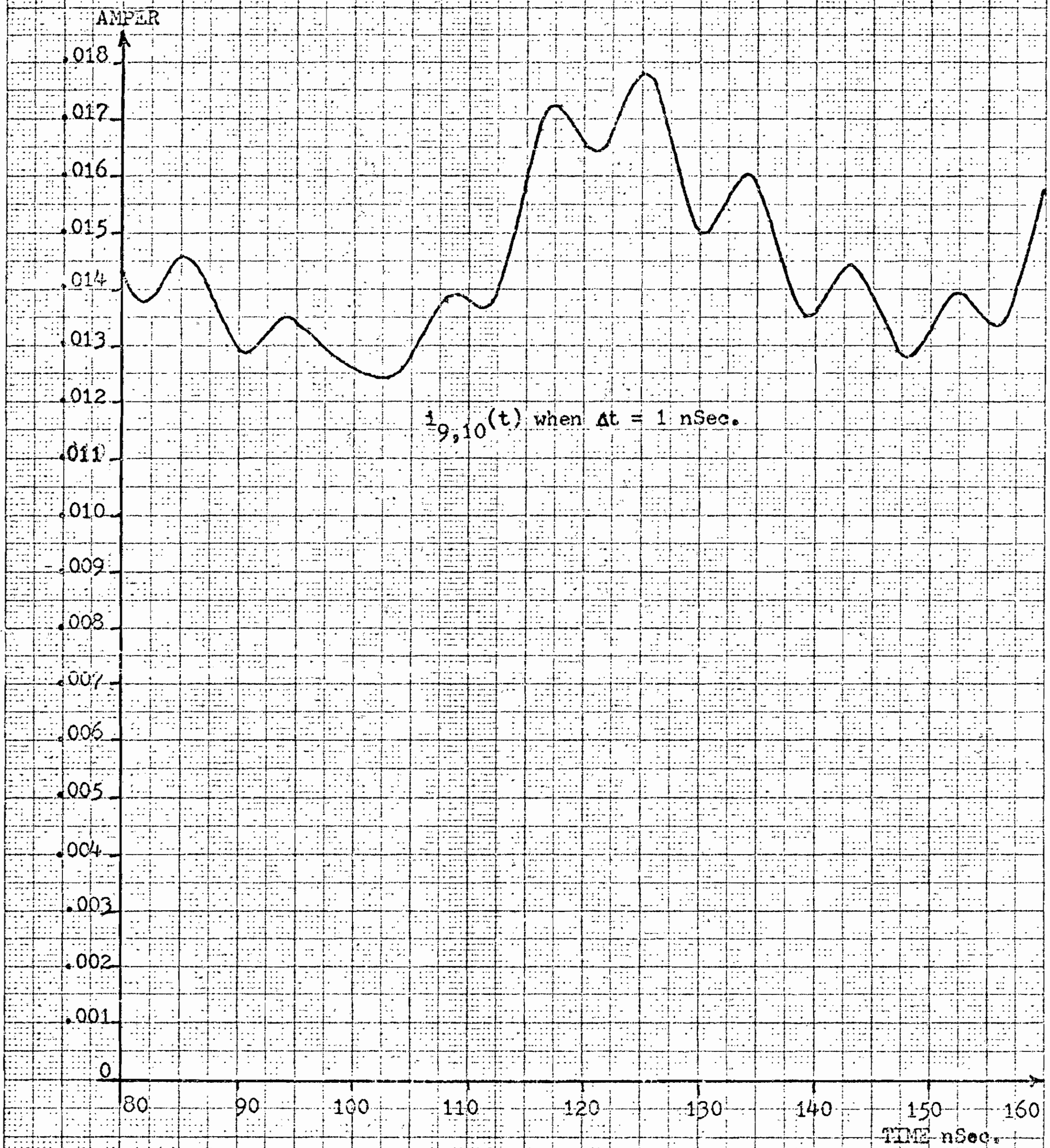


Fig. (36) (continued)

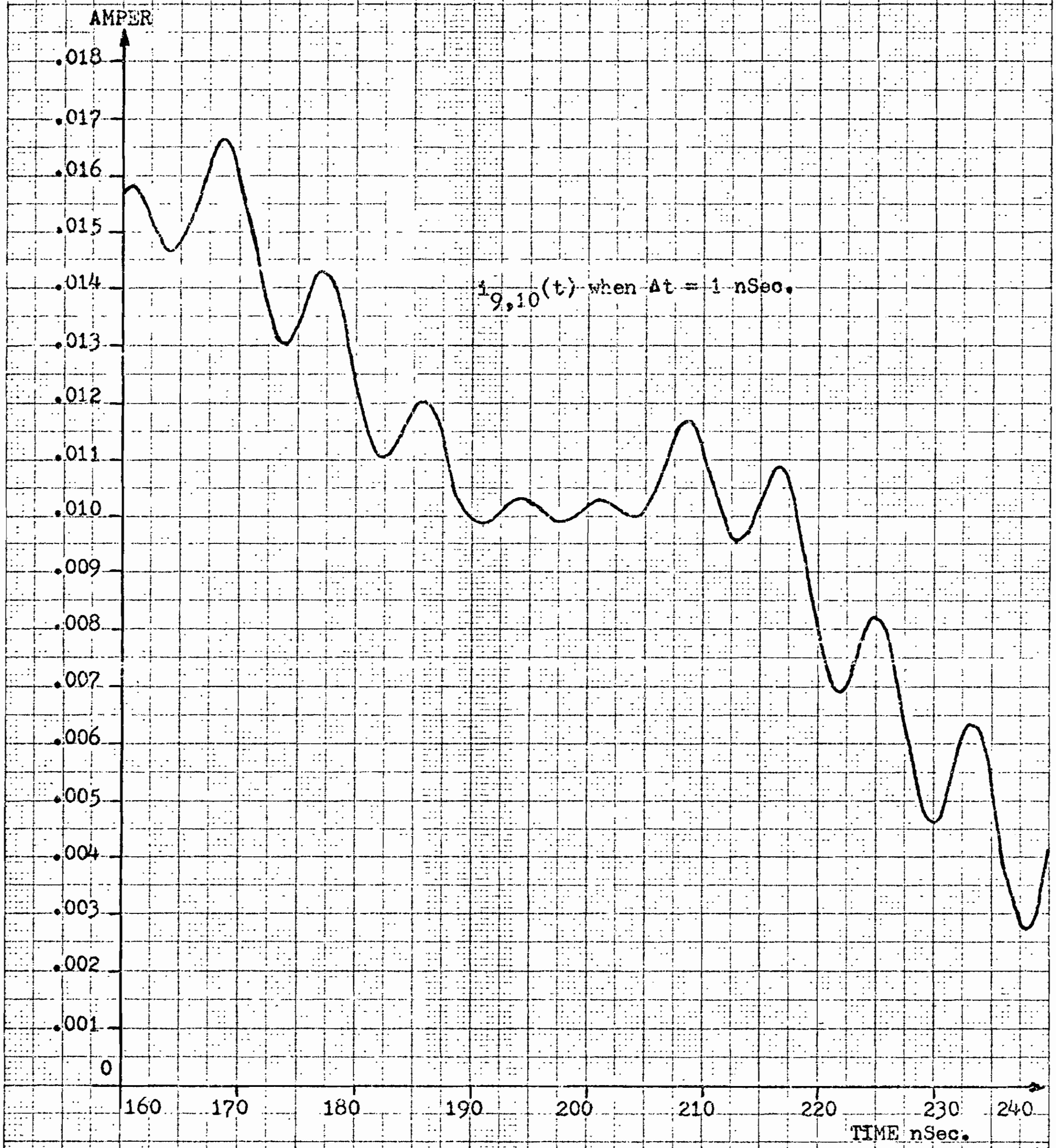


Fig. (36) (continued)

Voltage at node 10 ($e_{10}(t)$) and current from node 9 to node 10 ($i_{9,10}(t)$) were plotted in fig. (35), where first t was selected 1 nSec. and then 2 nSec. From these figures, the significance of selection of Δt appears. The peak value of $e_{10}(t)$ is 0.6849 Volt at t = 26 sec. and .5697 Volt at t = 26 Sec. when t is 1 nSec. and 2 n Sec., respectively. This shows that during time 26 nSec. and 28 nSec. that is t = 27 nSec., one point has been ignored in fig. (35) and this is because of selection of t = 2 nSec. This difference is the same for $i_{9,10}(t)$, which becomes clearer by studying the two curves.

From the above discussion, we note that the smaller Δt is the more accurate of the calculated data.

Nonlinear and Time-Varying Parameters

Nonlinear and time-varying parameters can be handled as well as linear parameters. However, when there is only one nonlinear parameter in a system, the solution is still linear and when there is more than one nonlinear parameter, the entire system becomes nonlinear and thus the solution gets very complicated.

Consider equation (61), when there is one nonlinear parameter, it is not included in matrix $[Y]$. To find an equation describing the relation of voltage and current, $i_{a,b}$ can be considered as two currents with two additional nodes:

$$i_b = i_{a,b} \text{ and } i_a = -i_{a,b}$$

The equation consists of two parts; linear and nonlinear. By superimposing the two additional currents $i_a = -i_b = -i_{a,b}$, equation (61) can be written in the form of:

$$e_U(t) = \underbrace{e_U(t)}_{\text{Linear part}} + \underbrace{Z \cdot i_{a,b}(t)}_{\text{nonlinear part}}$$

The linear part is computed by ignoring the nonlinear part. Matrix $\left[Z \right]$ is the precalculated difference of ath and bth columns of $\left[Y_{UU} \right]^{-1}$. By considering the two simultaneous equations derived by the linear network equation and the nonlinear equation, which is the characteristic of the nonlinear element, current $i_{a,b}(t)$ can be found as follows:

$$e_a(t) - e_b(t) = \underbrace{e_a(t) - e_b(t)}_{\text{linear part}} + \underbrace{(Z_a - Z_b)i_{a,b}(t)}_{\text{nonlinear part}} \quad (62)$$

Let us put equation (62) in a general form as follows:

$$e(t) = A - B\, i_{a,b}(t) \qquad (63)$$

The nonlinear equation in the form of given characteristics is:

$$e_a(t) - e_b(t) = f(i_{a,b}t)) \qquad (64)$$

The nonlinear characteristic can be presented point by point and by each set of points, a linear equation is made and all equations are plotted as piecewise linear; then the result will be the characteristic of the nonlinear element shown as a curve. This is represented by Bonneville Power Administration's program.

Now consider fig. (37), where the characteristic of a nonlinear element as piecewise linear and a linear network equation in the form of equation (63) are plotted. The intersection of these two curves gives the value of $i_{a,b}(t)$. For each step, when t changes the linear network line moves parallel with the latter one and every time a new value is found for $i_{a,b}(t)$. When $i_{a,b}(t)$ is known, it can be used in equation (62) and solved for e(t).

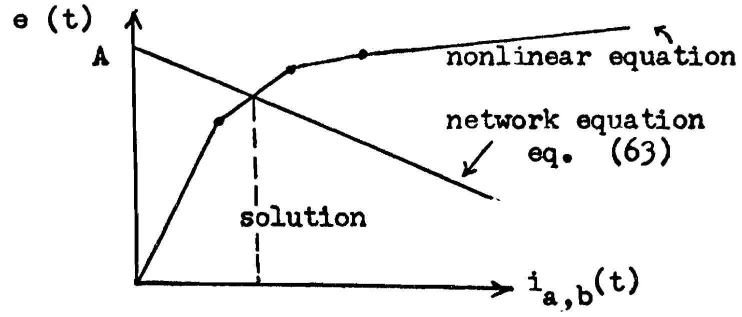


Fig. (37) Solution for nonlinear parameter.

A nonlinear characteristic can represent any type of nonlinear element. In the case of a lightning arrester, since there should be a certain voltage ($V_{breakdown}$) until the current be discharged, $i_{a,b}(t)$ remains zero until voltage breakdown and in the case of a system with one lightning arrester, the entire system is really linear until voltage breakdown.

When a nonlinear element is a time varying resistance, equation (64) becomes simpler. Since resistance is a function of time, equation (64) can be written in the following form:

$$e_a(t) - e_b(t) = R(t_R) \cdot i_{a,b}(t)$$

When a nonlinear element is an inductance, the characteristic is usually defined as:

$$\Psi = f(i_{a,b})$$

and the total flux is:

$$\Psi(t) = (e_a(t) - e_b(t))dt + \Psi(0) \qquad (65)$$

Now when the trapezoidal rule of integration is applied to equation (65), it gives:

$$e_a(t) - e_b(t) = (2/\Delta t)f(i_{a,b}(t)) - C(t - \Delta t),$$

$C(t - \Delta t)$ can be considered as initial condition or past history and for time zero and $(t - \Delta t)$, it is respectively as follows:

$$C(0) = (2/\Delta t)\Psi(0) + e_a(0) - e_b(0)$$

$$C(t - \Delta t) = C(t - 2\Delta t) + 2(e_a(t - \Delta t) - e_b(t - \Delta t)) \quad (66)$$

When a system consists of nonlinear elements, the time of study may not be the same as the time-varying of nonlinear elements. Therefore, it may be more than one time-varying and this makes the program longer.

In order to see how a system with one nonlinear element can be solved by digital computer, an example is illustrated on the next page.

Test Case No. 3

Consider a system having a nonlinear element (lightning arrester) shown on fig. (38). A lightning stroke hits a line close to a substation, which can be considered as a current impulse with a characteristic as shown in fig. (39). The voltages at nodes 1, 2, 3, 4, 8, and discharge current at lightning arrester are to be found.*

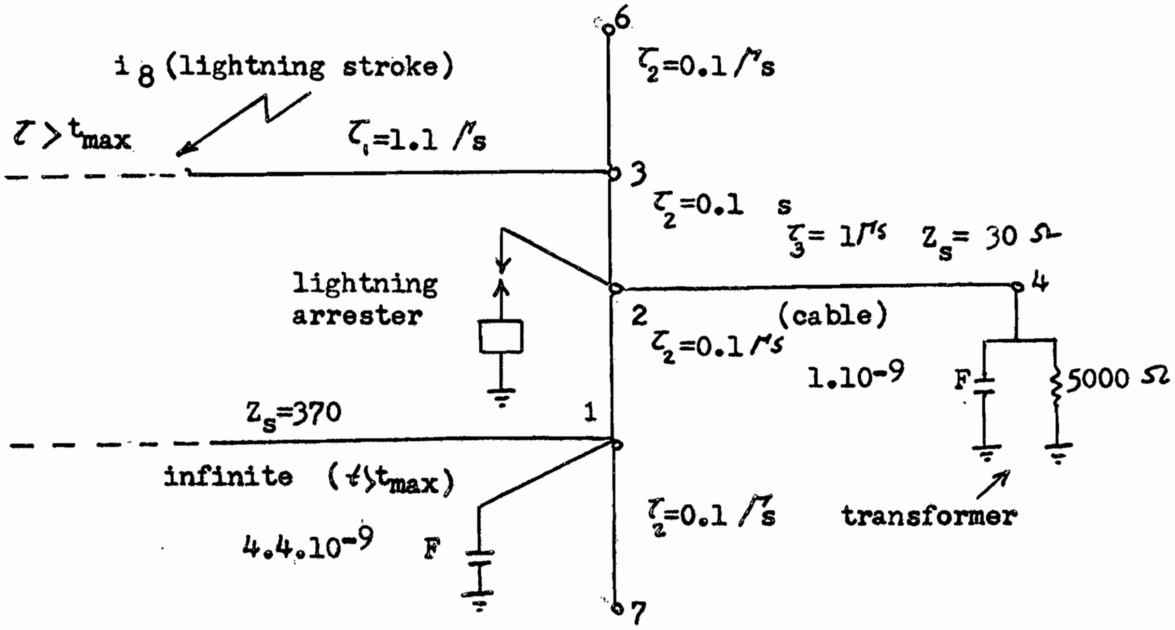


Fig. (38)

Given:

Z_s = 370Ω all lines except the line from 2 to 4 which is 30 ohm

t = time of study = .1 μsec., which is the smallest wave travel time on line. This can be smaller, but not more than .1 sec.

*This problem is taken from Hermann W. Dammel, Habilitation Thesis, submitted to the Munich Institute of Technology, May, 1970, p. 37.

Characteristics of the lightning arrester:

$V_{breakdown} = 610$ V

current i in KA	0	.5	1	1.5	2.5	3	10
voltage e in KV	0	440	510	540	580	590	660

Characteristic of the lightning surge is shown in fig. (39).

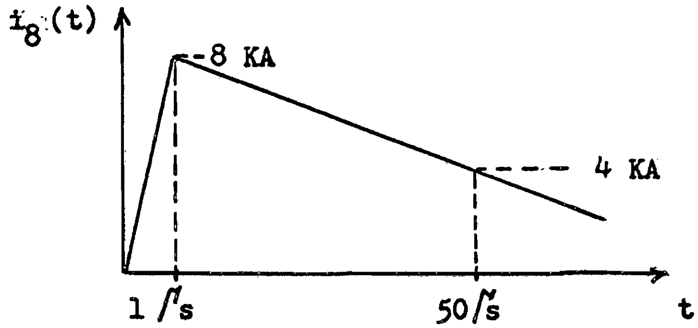


Fig. (39)

Solution:

The equivalent impedance network of the system is shown on the next page.

Let: τ = wave travel time on a line

Δt = time of study

The node equations can be written as follows:

Node 8:

$$i_1(t) + i_{83}(t) - i_8(t) = 0$$
$$(1/370)e_8(t) + (1/370)e_8(t) + I_{A8}(t - \tau_1) - i_8(t) = 0$$
$$e_8(t) = (370/2)(i_8(t) - I_{A8}(t - \tau_1))$$

Node 3:

$$i_{38}(t) + i_{32}(t) + i_{36}(t) = 0$$
$$(1/370)e_3(t) + I_{B3}(t - \tau_1) + (1/370)e_3(t) + I_{A3}(t - \tau_2) + (1/370)$$
$$e_3(t) + I_{C3}(t - \tau_2) = 0$$

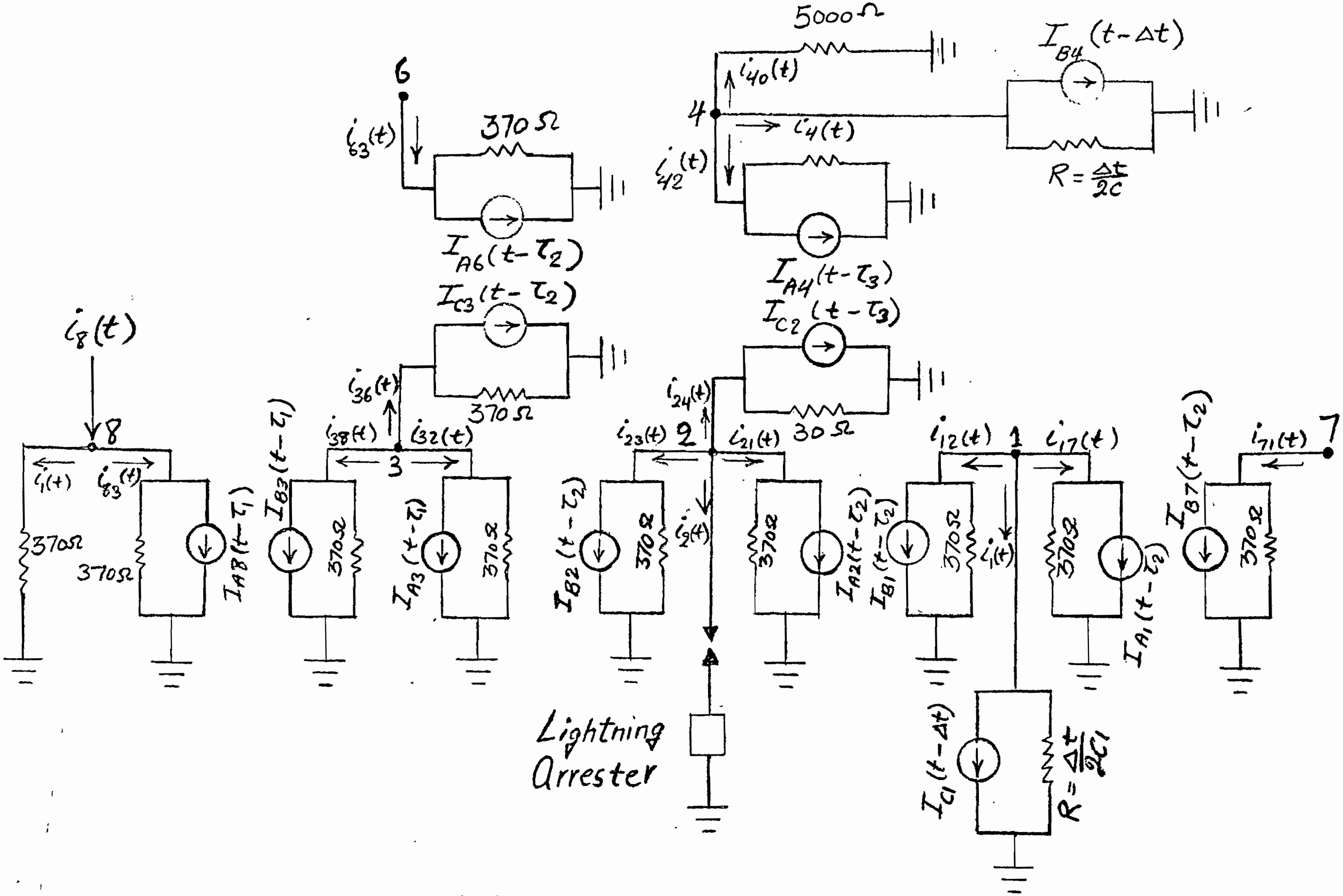


Fig. (40) Eqivalent inpedance network

$e_3(t) = (-370/3)(I_{B3}(t - \tau_1) + I_{A3}(t - \tau_3) + I_{C3}(t - \tau_3))$

Node 2:

Before $V_{breakdown}$ = 610 V:

$i_{23}(t) + i_{21}(t) + i_{24}(t) = 0$

$$(1/370)e_2(t) + I_{B2}(t - \tau_2) + (1/370)e_2(t) + I_{A2}(t - \tau_2) + (1/30)e_2(t) + I_{C2}(t - \tau_3) = 0$$

$e_2(t) = (-1110/43)(I_{B2}(t - \tau_2) + I_{A2}(t - \tau_2) + I_{C2}(t - \tau_3))$

At and after $V_{breakdown}$:

$i_{23}(t) + i_{21}(t) + i_{24}(t) + i_2(t) = 0$

$$e_2(t) = (-1110/43)(\underbrace{I_{B2}(t - \tau_2) + I_{A2}(t - \tau_2) + I_{C2}(t - \tau_3)}_{\text{linear part}} - i_8(t)) \qquad (67)$$

This also can be written in the following form:

$$e_2(t) = e_{L2}(t) - 1110/43\ i_8(t) \qquad (68)$$

Node 1:

$i_{12}(t) + i_1(t) + i_{17}(t) = 0$

$$(1/370)e_1(t) + I_{B1}(t - \tau_2) + (2C/\Delta t)e_1(t) + I_{C2}(t - \tau_3) + (1/370)e_1(t) + I_{A1}(t - \tau_2) = 0$$

$e_1(t) = (-1/(2/370*2C/\Delta t))\ (I_{B1}(t - \tau_2) + I_{C1}(t - \Delta t) + I_{A1}(t - \tau_2))$

Node 4:

$i_{42}(t) + i_4(t) + i_{40}(t) = 0$

$$(1/30)e_4(t) + I_{A4}(t - \tau_3) + (2C/\Delta t)e_4(t) + I_{B4}(t - \Delta t) + (1/5000)e_4(t) = 0$$

$e_4(t) = (-1/(1/30+2C/\Delta t+1/5000))(I_{A4}(t - \tau_3) + I_{B4}(t - \Delta t))$

Nodes 6 and 7:

$e_6(t) = -370\ I_{A6}(t - \tau_2)$

$e_7(t) = -370\ I_{B7}(t - \tau_2)$

Considering the lightning surge characteristics, in fig. (39), current $i_8(t)$ can be determined in the following form:

$i_8(t) = 8t$ for $0 < t \leqslant 1$

$i_8(t) = (-4/49)t + 396/49$ for $1 < t < 50$

The characteristic curve of the lightning arrester is plotted in fig. (41) by having 7 points given in characteristic of the lightning arrester. Equation of the line between each two points is as follows:

$e_2(t) = 880\ i_2(t)$ for $0 \leqslant e_2(t) \leqslant 440$

$e_2(t) = 370 + 140\ i_2(t)$ for $440 \leqslant e_2(t) \leqslant 510$

$e_2(t) = 450 + 60\ i_2(t)$ for $510 \leqslant e_2(t) \leqslant 540$

$e_2(t) = 480 + 40\ i_2(t)$ for $540 \leqslant e_2(t) \leqslant 580$

$e_2(t) = 530 + 20\ i_2(t)$ for $580 \leqslant e_2(t) \leqslant 590$

$e_2(t) = 560 + 10\ i_2(t)$ for $590 \leqslant e_2(t) \leqslant 660$

Now consider equation (68) which is:

$$e_2(t) = e_{L2}(t) - (1110/43)\ i_8(t)$$

Let us find the intersection of this line with nonlinear segments of the lightning arrester as shown in fig. (41).

1) $i_2(t) = e_{L2}(t)/880 + 1110/43)$ for $i_2(t) \leqslant .5$

2) $i_2(t) = (e_{L2}(t) - 370)/(140 + 1110/43)$ for $i_2(t) \leqslant 1.0$

3) $i_2(t) = (e_{L2}(t) - 450)/(60 + 1110/43)$ for $i_2(t) \leqslant 1.5$

4) $i_2(t) = (e_{L2}(t) - 480)/(40 + 1110/43)$ for $i_2(t) \leqslant 2.5$

5) $i_2(t) = (e_{L2}(t) - 530)/(20 + 1110/43)$ for $i_2(t) \leqslant 3.0$

6) $i_2(t) = (e_{L2}(t) - 560)/(10 + 1110/43)$ for $i_2(t) \leqslant 10$

As was mentioned before, the system is linear up to voltage breakdown and therefore $e_2(t) = e_{L2}(t)$. Once $e_{L2}(t)$ reaches $V_{breakdown}$, $i_2(t)$ is computed by first equation derived from the intersection of linear equation and nonlinear segments as above. If the computed value

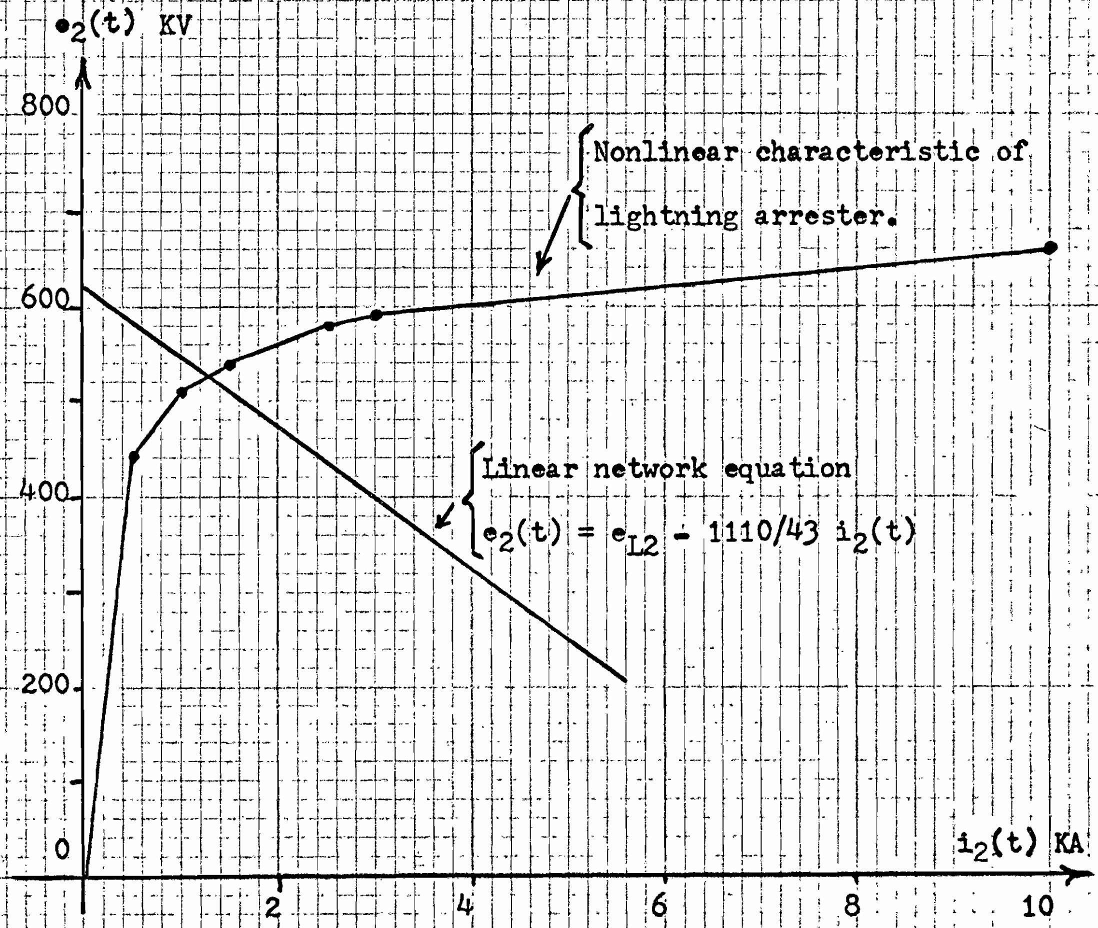


Fig. (41). Characteristic curve of the lightning arrester.

of $i_2(t)$ corresponds to the given value of $i_2(t)$, it is accepted, otherwise the next equation should be used. For example, if the first equation is used, the computed value of $i_2(t)$ is $\leqslant .5$, $i_2(t)$ is accepted, otherwise the second or further equations should be used. The same procedure should be followed for all equations until the computed value of $i_2(t)$ meets the corresponding equation condition.

Once an accepted computed $i_2(t)$ is found, $e_2(t)$ can be computed by equation (67). For the next step the same procedure is used and $e_2(t)$ does not have to reach or to be more than voltage breakdown, because after $V_{breakdown}$ the lightning arrester still discharges current.

The past history currents I_{A8}, I_{B3}, I_{A3}, I_{A2}, I_{B1}, I_{A1}, I_{A1}, I_{C1}, I_{A6}, I_{A4}, and I_{B7} are computed by means of equations (35), (37), and (56).

Now all equations are set and we are ready to write a digital computer program.

Let us define the symbols used in the program.

D = Δt

T1 = maximum time

C1 and C = capacitance

E(8,I) = $e_8(t)$

E(2,I) = $e_2(t)$

. . . .

. . . .

A(8,I) = $I_{A8}(t)$

B(3,I) = $I_{B3}(t)$

C(2,I) = $I_{C2}(t)$

$$R(8,I) = i_{83}(t)$$

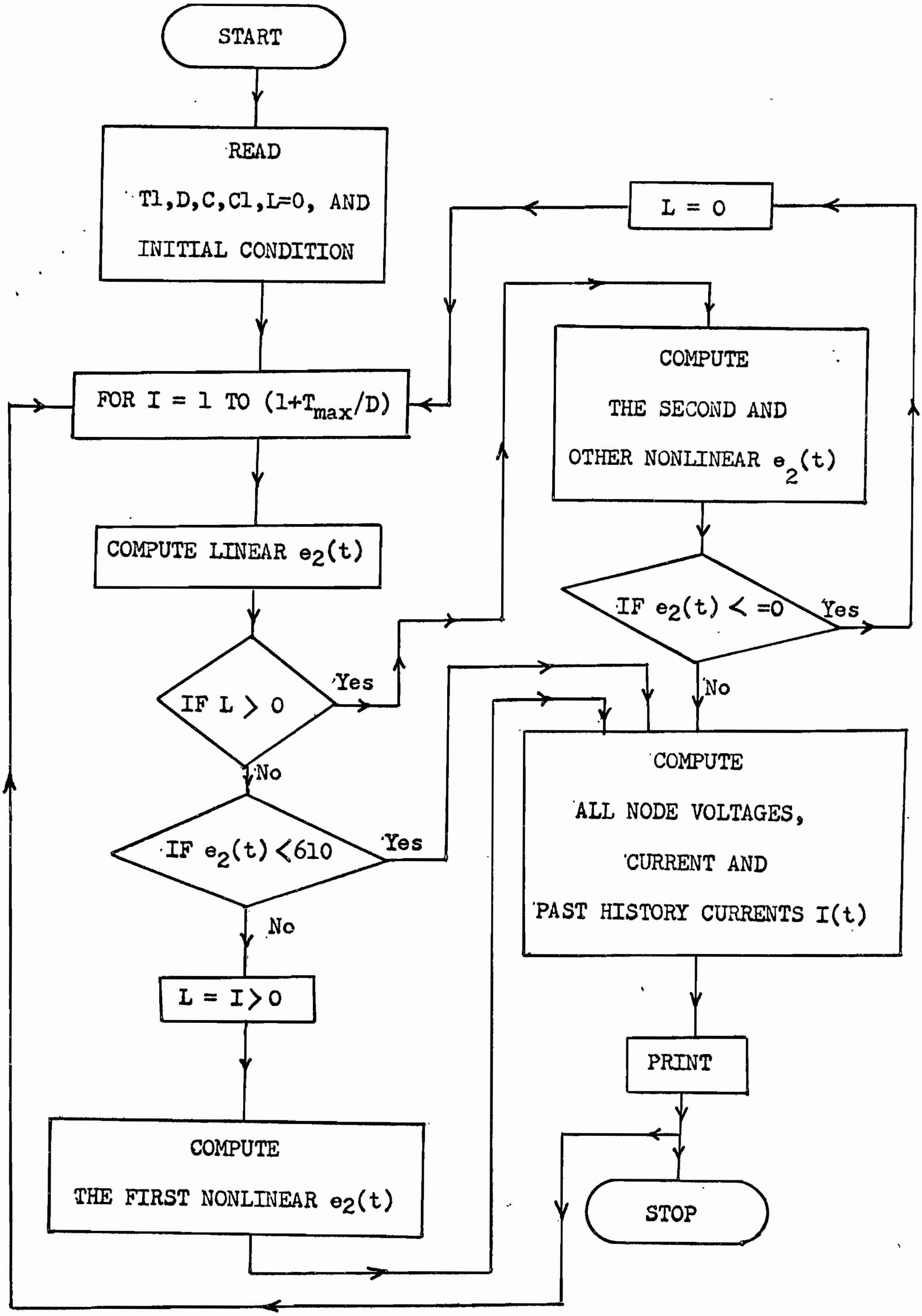
$$P(3,I) = i_{38}(t)$$

$$I8 = i_8(t)$$

$$I2 = i_2(t)$$

L is a reference for first nonlinear computed $e_2(t)$.

A flow chart and digital computer program are on the next page.


START
READ
T1,D,C,C1,L=0, AND
INITIAL CONDITION
FOR I = 1 TO (1+T_max/D)
COMPUTE LINEAR e2(t)
IF L > 0
Yes
No
IF e2(t) < 610
Yes
No
L = I > 0
COMPUTE
THE FIRST NONLINEAR e2(t)
L = 0
COMPUTE
THE SECOND AND
OTHER NONLINEAR e2(t)
IF e2(t) < =0
Yes
No
COMPUTE
ALL NODE VOLTAGES,
CURRENT AND
PAST HISTORY CURRENTS I(t)
PRINT
STOP

Computer program:

```
100 DIM E(8,100),A(8,100),B(8,100),C(8,100)
105 DIM R(8,100),P(8,100),Q(8,100)
110 READ T1,D
120 READ C,C1
130 READ A(8,1),R(8,1),B(3,1),P(3,1),Q(3,1),R(3,1),B(4,1)
140 READ C(3,1),A(3,1),B(2,1),P(2,1),R(2,1),Q(2,1),Q(4,1)
150 READ C(2,1),A(2,1),B(1,1),P(1,1),Q(1,1),R(1,1),P(4,1)
160 READ A(1,1),C(1,1),B(7,1),P(7,1),P(6,1),A(6,1),A(4,1)
165 PRINT "TIME","VOLTAGE AT 4","VOLTAGE AT 8","DISCHARGE CURRENT AT 2"
166 PRINT
170 L = 0
180 I1 = 1 + T1/D
190 FOR I = 2 TO I1
200 T = D*(I - 1)
210 IF T>1. THEN 240
220 I8 = 8*T
230 GO TO 250
240 I8 = -(4/49)*T + 396/49
250 K = I - 11
260 N = I - 1
270 M = I - 10
280 IF T>1. THEN 320
290 K = 1
300 N = 1
```

```
310 M = 1
320 E(8,I)=(370/2)*(I8-A(8,K))
330 E(3,I)=(-370/3)*(B(3,K)+A(3,N)+C(3,N))
340 P(3,I)=(1/370)*E(3,I)+B(3,K)
350 A(8,I)=-(1/370)*E(3,I)-P(3,I)
360 R(8,I)=(1/370)*E(8,I)+A(8,K)
370 B(3,I)=-(1/370)*E(8,I)-R(8,I)
380 E(2,I)=(-1110/43)*(B(2,N)+A(2,N)+C(2,M))
390 IF L>0 THEN 420
400 IF E(2,I)<610 THEN 660
410 L = 1
420 I2=E(2,I)/880+1110/43)
430 IF I2 > .5 THEN 450
440 GO TO 580
450 I2=(E2,I)-370)/(140+1110/43)
460 IF I2 >1. THEN 480
470 GO TO 580
480 I2=(E(2,I)-450)/(60 + 1110/43)
490 IF I2> 1.5 THEN 510
500 GO TO 580
510 I2=(E(2,I)-480)/(40+1110/43)
520 IF I2> 2.5 THEN 540
530 GO TO 580
540 I2=(E(2,I)-530)/(20+1110/43)
550 IF I2>3. THEN 570
560 GO TO 580
```

```
570 I2=(E(2,I)-560)/(10+1110/43)
580 E(2,I)=(-1110/43)*(B(2,N)+A(2,N)+C(2,M)+I2)
590 IF E(2,I)>.0 THEN 660
600 L = 0
610 GO TO 390
660 P(2,I)=(1/370)*E(2,I)+B(2,N)
670 A(3,I)=-(1/370)*E(2,I)-P(2,I)
680 R(3,I)=(1/370)*E(3,I)+A(3,N)
690 B(2,I)=-(1/370)*E(3,I)-R(3,I)
700 E(6,I)=-370*A(6,N)
710 Q(3,I)=(1/370)*E(3,I)+C(3,N)
720 A(6,I)=-(1/370)*E(3,I)-Q(3,I)
730 P(6,I)=(1/370)*E(6,I)+A(6,N)
740 C(3,I)=-(1/370)*E(6,I)-P(6,I)
750 E(4,I)=(-1/(1/30+2*C/1E-7+1/5000))*(A(4,M)+B(4,N))
760 P(4,I)=(1/30)*E(4,I)+A(4,M)
770 C(2,I)=-(1/30)*E(4,I)-P(4,I)
780 Q(2,I)=(1/30)*E(2,I)+C(2,M)
790 A(4,I)=-(1/30)*E(2,I)-Q(2,I)
800 Q(4,I)=(2*C/1E-7)*E(4,I)+B(4,N)
810 B(4,I)=-Q(4,I)-(2*C/1E-7)*E(4,I)
820 E(1,I)=(-1/(2/370+2*C1/1E-7))*(B(1,N)+C(1,N)+A(1,N))
830 P(1,I)=(1/370)*E(1,I)+B(1,N)
840 A(2,I)=-(1/370)*E(1,I)-P(1,I)
850 R(2,I)=(1/370)E(2,I)+A(2,N)
860 B(1,I)=-(1/370)*E(2,I)-R(2,I)
```

```
870 Q(2,I)=(2*Cl/1E-7)*E(1,I)+Cl,N)
880 C(1,I)=-Q(1,I)-(2*Cl/1E-7)*E(1,I)
890 E(7,I)=-370*B(7,N)
900 P(7,I)=(1/370)E(7,I)+B(7,N)
910 A(1,I)=-(1/370)*E(7,I)-P(7,I)
920 R(1,I)=(1/370)*E(1,I)+A(1,N)
930 B(7,I)=-(1/370)*E(1,I)-R(1,I)
940 PRINT T,E(4,I),E(8,I),I2
950 NEXT I
960 DATA 8,.1
970 DATA 1E-9,4.4E-9
980 DATA .0,.0,.0,.0,.0,.0,.0
985 DATA .0,.0,.0,.0,.0,.0,.0
990 DATA .0,.0,.0,.0,.0,.0,.0
995 DATA .0,.0,.0,.0,.0,.0,.0
999 END
```

The result is printed on the next page.

TIME	VOLTAGE AT 1	VOLTAGE AT 2	VOLTAGE AT 3
0.1	0	0	0
0.2	0	0	0
0.3	0	0	0
0.4	0	0	0
0.5	0	0	0
0.6	0	0	0
0.7	0	0	0
0.8	0	0	0
0.9	0	0	0
1	0	0	0
1.1	0	0	0
1.2	0	0	98.6667
1.3	0	13.7674	197.333
1.4	0.796727	27.5349	305.178
1.5	3.09469	52.6196	413.023
1.6	8.17181	77.9137	406.036
1.7	17.1702	95.7314	399.188
1.8	30.5749	114.267	391.841
1.9	48.6826	127.957	385.113
2	71.2459	142.908	408.447
2.1	97.8387	159.182	433.054
2.2	127.88	177.105	355.806
2.3	160.689	182.25	280.291
2.4	194.686	189.212	187.119
2.5	228.379	184.665	95.8327
2.6	259.62	181.648	121.981
2.7	286.477	186.377	149.782
2.8	307.718	191.829	180.961
2.9	322.449	202.553	213.013
3	330.441	213.011	214.599
3.1	331.831	222.078	215.938
3.2	327.052	229.999	218.74
3.3	316.82	242.073	220.308
3.4	302.395	274.432	233.874
3.5	286.432	304.268	260.281
3.6	271.456	347.879	287.287
3.7	260.767	382.929	337.411
3.8	256.943	401.608	376.146
3.9	261.231	423.078	408.871
4	274.619	435.337	437.549
4.1	297.042	453.264	441.466
4.2	328.248	472.31	454.818
4.3	367.549	490.177	464.781
4.4	413.735	487.468	479.084
4.5	464.032	491.87	488.67
4.6	515.914	481.781	499.065
4.7	565.884	484.684	487.212
4.8	611.293	504.415	487.314
4.9	650.716	524.373	490.547
5	683.037	553.991	502.323

TIME	VOLTAGE AT 1	VOLTAGE AT 2	VOLTAGE AT 3
5.1	708.101	579.998	539.173
5.2	725.963	604.708	569.782
5.3	737.013	569.382	604.924
5.4	738.546	584.556	594.785
5.5	731.512	599.494	609.091
5.6	717.151	607.914	585.296
5.7	696.547	617.804	589.312
5.8	671.435	622.544	609.243
5.9	643.676	624.226	613.248
6	614.896	625.106	628.895
6.1	586.583	625.315	631.302
6.2	560.38	624.938	625.919
6.3	537.915	625.452	624.919
6.4	520.43	623.154	620.186
6.5	508.601	618.682	617.6
6.6	502.9	615.56	616.755
6.7	503.653	612.78	613.476
6.8	510.59	613.197	610.219
6.9	523.173	619.353	609.278
7	541.126	623.485	611.81
7.1	563.827	631.564	615.936
7.2	590.368	637.167	626.512
7.3	619.414	637.983	633.315
7.4	649.469	616.838	638.401
7.5	677.781	625.272	626.664
7.6	703.221	623.342	629.305
7.7	724.059	622.709	611.654
7.8	739.131	621.63	615.16
7.9	747.717	623.038	617.066
8	749.363	625.572	616.703

TIME	VOLTAGE AT 4	VOLTAGE AT 8	DISCHARGE CURRENT AT 2
0.1	0	148	0
0.2	0	296	0
0.3	0	444	0
0.4	0	592	0
0.5	0	740	0
0.6	0	888	0
0.7	0	1036	0
0.8	0	1184	0
0.9	0	1332	0
1	0	1480	0
1.1	0	1478.49	0
1.2	0	1476.98	0
1.3	0	1475.47	0
1.4	0	1473.96	0
1.5	0	1472.45	0
1.6	0	1470.94	0
1.7	0	1469.43	0
1.8	0	1467.92	0
1.9	0	1466.41	0
2	0	1464.9	0
2.1	0	1463.39	0
2.2	0	1461.88	0
2.3	17.145	1411.03	0
2.4	47.1007	1360.19	0
2.5	87.9116	1318.53	0
2.6	140.333	1276.86	0
2.7	180.769	1120.36	0
2.8	215.818	964.004	0
2.9	247.09	807.147	0
3	274.852	650.909	0
3.1	306.719	524.733	0
3.2	341.249	399.829	0
3.3	361.247	322.581	0
3.4	371.27	247.066	0
3.5	371.743	153.895	0
3.6	362.203	62.6082	0
3.7	366.747	88.7566	0
3.8	378.277	116.558	0
3.9	395.506	147.737	0
4	417.529	179.788	0
4.1	436.277	181.375	0
4.2	452.693	182.714	0
4.3	469.236	185.515	0
4.4	496.023	187.084	0
4.5	526.961	200.649	0
4.6	557.24	227.056	0
4.7	585.594	254.062	0
4.8	596.603	304.187	0
4.9	601.359	342.922	0
5	604.312	375.646	0

TIME	VOLTAGE AT 4	VOLTAGE AT 8	DISCHARGE CURRENT AT 2
5.1	605.784	404.324	0
5.2	611.355	408.241	0
5.3	616.738	421.594	2.23454
5.4	611.947	431.556	2.72779
5.5	605.203	445.859	3.94935
5.6	601.695	455.445	4.79141
5.7	601.988	465.84	5.78043
5.8	622.763	453.987	6.25437
5.9	651.265	454.09	6.42265
6	683.295	457.322	6.5106
6.1	718.011	469.098	6.53151
6.2	749.758	505.949	6.49376
6.3	708.16	536.557	6.54518
6.4	670.589	571.699	6.31544
6.5	683.41	561.56	5.86816
6.6	671.122	575.866	5.55599
6.7	660.023	552.071	5.278
6.8	650.204	556.087	5.31972
6.9	648.591	576.019	5.93532
7	646.744	580.023	6.34849
7.1	645.275	595.67	7.15645
7.2	642.609	598.078	7.7167
7.3	636.603	592.695	7.7983
7.4	631.733	591.695	5.68377
7.5	630.943	586.961	6.52716
7.6	629.24	584.376	6.33418
7.7	625.73	583.531	6.27093
7.8	618.288	580.251	6.16304
7.9	600.743	576.994	6.3038
8	581.843	576.053	6.55724

The curves are shown on next page.

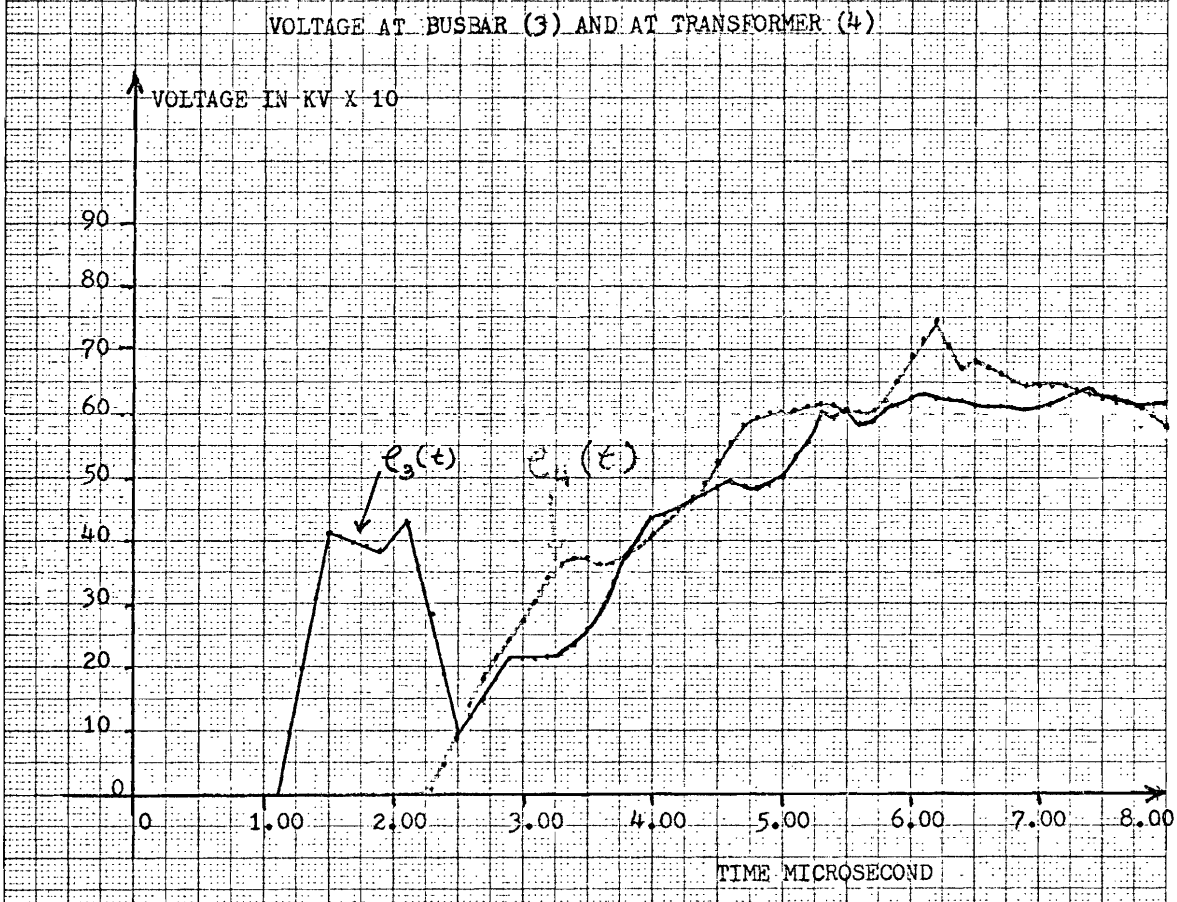

VOLTAGE AT BUSBAR (3) AND AT TRANSFORMER (4)
VOLTAGE IN KV X 10
90
80
70
60
50
40
30
20
10
0
0
1.00
2.00
3.00
4.00
5.00
6.00
7.00
8.00
TIME MICROSECOND
$e_3(t)$
$e_4(t)$

Fig. (42)

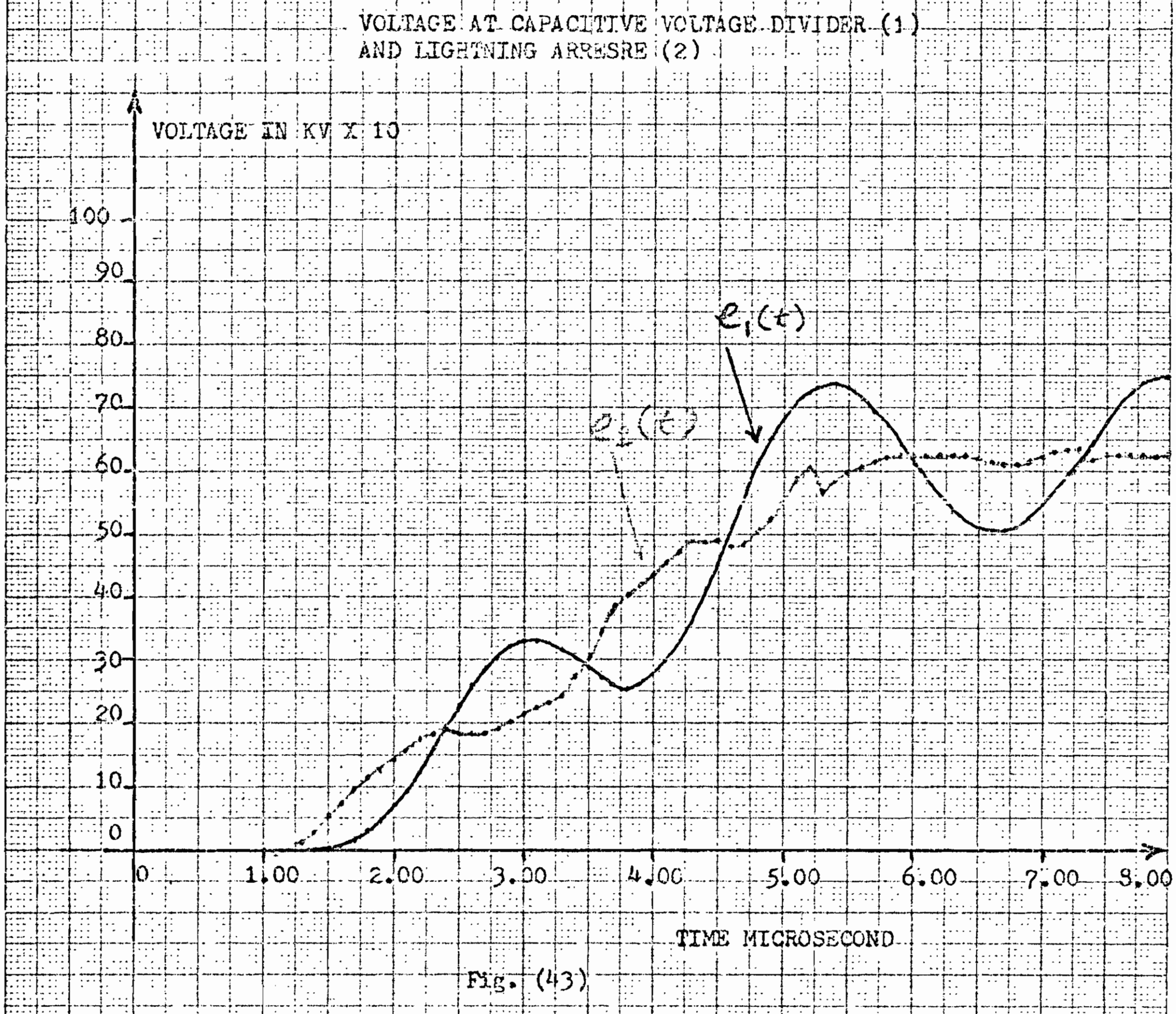

VOLTAGE AT CAPACITIVE VOLTAGE DIVIDER (1)
AND LIGHTNING ARRESRE (2)
VOLTAGE IN KV X 10
100
90
80
70
60
50
40
30
20
10
0
0
1.00
2.00
3.00
4.00
5.00
6.00
7.00
8.00
TIME MICROSECOND
$e_1(t)$
$e_2(t)$

Fig. (43)

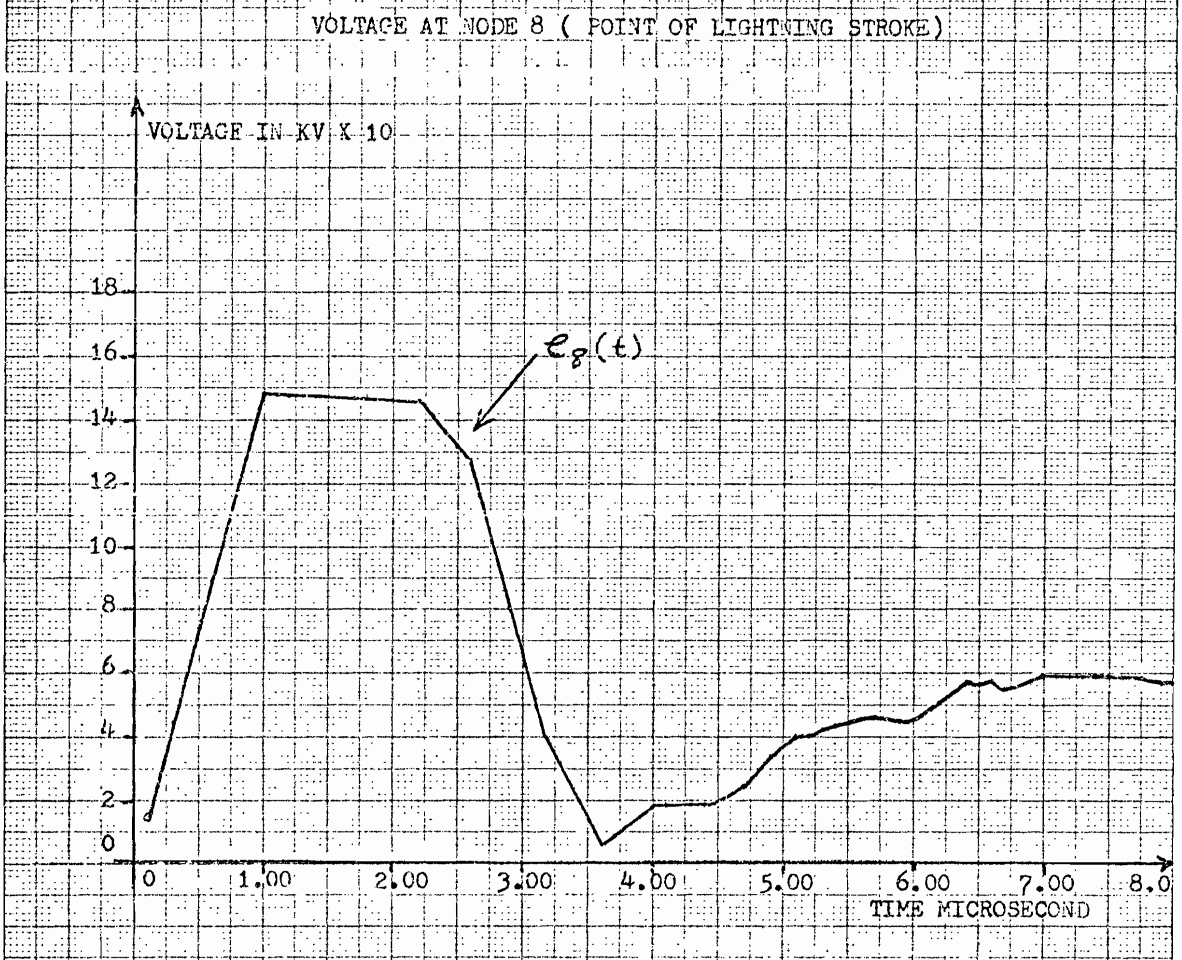

VOLTAGE AT NODE 8 (POINT OF LIGHTNING STROKE)
VOLTAGE IN KV X 10
18
16
14
12
10
8
6
4
2
0
$e_8(t)$
0
1.00
2.00
3.00
4.00
5.00
6.00
7.00
8.0
TIME MICROSECOND

Fig. (44)

DISCHARGE CURRENT IN LIGHINING ARRESTER

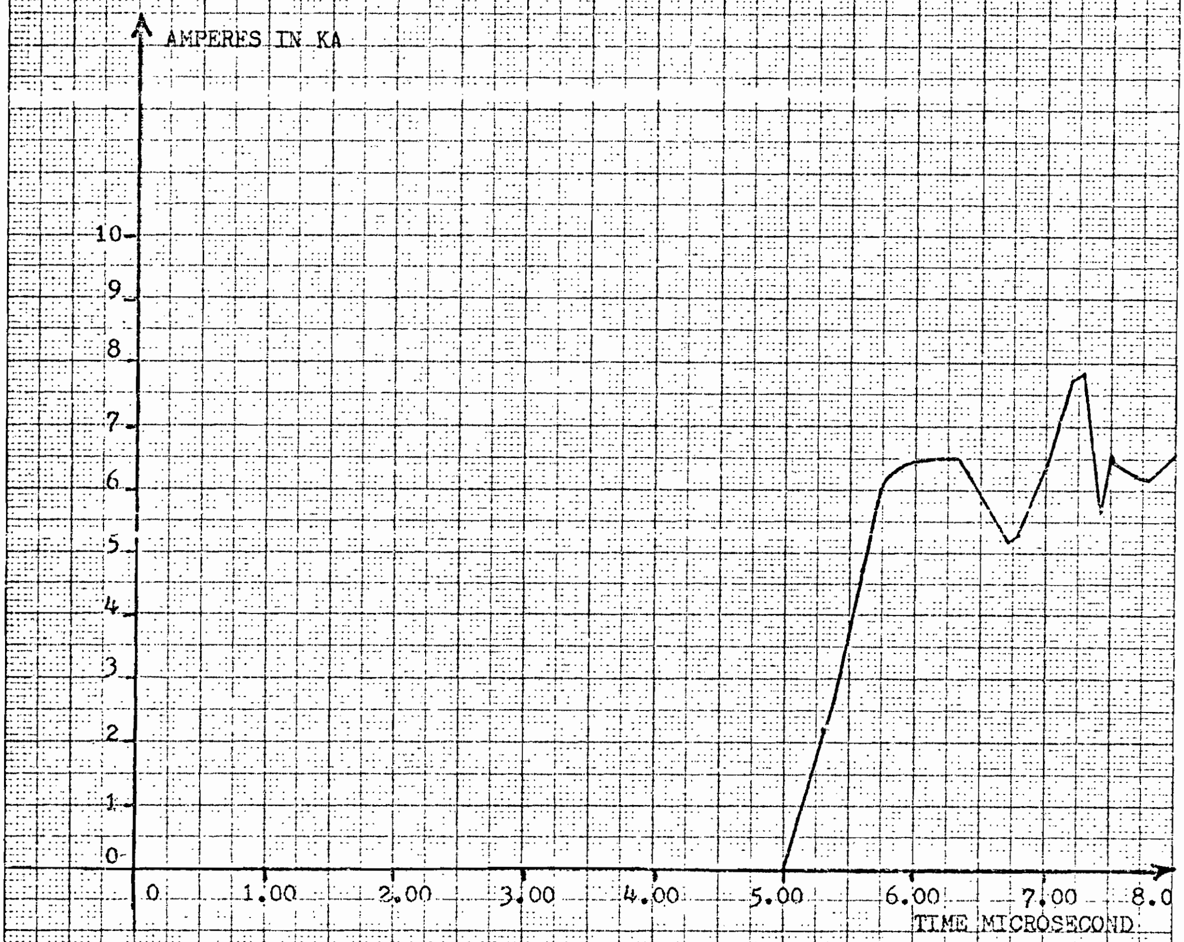


Fig. (45)

Past History Current I

In the last three examples, we saw that the past history plays an important role in digital computer solution. For each step of time the equivalent current source I has to be recorded in order to build I_{total} for the next step and for each inductance and capacitance, we need to know $I_{a,b}(t - \Delta t)$. This requires a double list of I_a and I_b. Now if the currents in equations (50) and (55) be expressed by equations (49) and (55), respectively, we may find an equation describing $I_{a,b}(t - \Delta t)$ in a form in which I may be computed faster.

Let us write (49) and (50) again for the sake of convenience.

$$i_{a,b}(t) = (\Delta t/2L)(e_a(t) - e_b(t)) + I_{a,b}(t - \Delta t) \qquad (69)$$

and at time $(t - \Delta t)$ equation (69) can be written in the form of:

$$i_{a,b}(t - \Delta t) = (\Delta t/2L)(e_a(t- \Delta t) - e_b(t- \Delta t)) + I_{a,b}(t - 2 \Delta t),$$

equation (50) is:

$$I_{a,b}(t-\Delta t) = i_{a,b}(t-\Delta t) + (\Delta t/2L)(e_a(t-\Delta t) - e_b(t-\Delta t)).$$

Find $i_{a,b}(t - \Delta t)$ from equation (50) and substitute in equation (69) and reorder it; we have:

$$I_{a,b}(t- \Delta t) = I_{a,b}(t- \Delta t) + 2(\Delta t/2L)(e_a(t- \Delta t) - e_b(t- \Delta t)) \qquad (70)$$

Similarly, equations (55) and (56) give:

$$I_{a,b}(t-\Delta t) = -I_{a,b}(t-2\Delta t) - 2(2C/\Delta t)(e_a(t- \Delta t) - e_b(t- \Delta t)) \qquad (71)$$

Equations (70) and (71) could be written in a general form as follows:

$$I_{a,b}(t - \Delta t) = \pm (I_{a,b}(t - 2\Delta t) + 2H(t)). \qquad (72)$$

Where:

+ is for inductance

- is for capacitance

$H(t) = (\Delta t/2L)(e_a(t- \Delta t) - e_b(t- \Delta t))$ for inductance

$H(t) = (2C/\Delta t)(e_a(t- \Delta t) - e_b(t- \Delta t))$ for capacitance

Accuracy

Since approximation is made by trapezoidal rule of integration for lumped parameters, there is some error in computing voltage and current. However, as long as Δt is selected sufficiently small, the error in practice is completely ignorable. The result obtained by trapezoidal method is adequate for the purpose of digital computer solution. Compared with other methods, the approximation is more or as accurate as other alternative methods. As long as the oscillation of highest frequency is represented by sufficient number of points, the selection of Δt is not too important.

Before we draw our conclusion, it would be a good idea to discuss Laplace transformation technique of the solution of electromagnetic transient or steady state in a circuit. This will show the advantages and disadvantages of both techniques.

LAPLACE TRANSFORMATION TECHNIQUE

Differential equations have been used traditionally to describe engineering and physical problems particularly in the electrical field. Since a general solution is not always available for differential equations, the Laplace transformation is a mathematical tool that greatly facilitates the solution of constant-coefficient linear differential equations. By Laplace transformation, a differential equation can be transferred into relatively simple algebraic equations. The complete solution of the original differential equation is obtained by transformation back.

Laplace transformation eliminates the independent variable in differential equations and operator s takes its place. The s operator is a complex quantity and may be handled algebraically in an equation.

The transformation from the real independent variable domain to the s complex variable domain is obtained by integration of the differential equation as follows:

By definition the Laplace transformation integral is:

$$F(s) = \int_{0^+}^{\infty} e^{-st} f(t)dt = \mathcal{L} f(t)$$

where s is called Laplace operator, f(t) is a known function and F(s) is the Laplace form of function f(t).

The inverse transformation or transformation back from s domain to the t domain is:

$$f(t) = (1/2\pi j) \oint F(s)e^{st}ds$$

Of course there are numbers of books which describe the Laplace transformation and its tables to which the reader may refer for more information.*

In order to compare the first method with the Laplace transformation technique, an example is illustrated on the next page.

Laplace transformation result is shown in figs. (47), (48), and (49). The digital computer solution result is shown in fig. (50).

The two results are fairly similar to each other; the difference could be considered as the approximation used in computing voltage and current in both Laplace and digital computer methods. The two results are individually accepted for practical purposes.

*Floyd E. Nixon, <u>Handbook of Laplace Transformation</u>, 2d ed. (Englewood Cliffs, N. J.: 1965), pp. 21-44.

Test case No. 4

Consider the two R - L - C branches shown below. This is similar to the example of test case No. 2 with only two branches.

Let us find voltage at node 2.

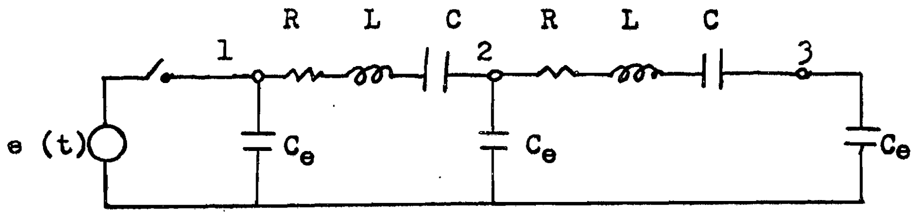


Fig. (46)

Given:

$e_1(t) = 1.0$ for $t > 0$

$e_1(t) = 0.$ for $t \leqslant 0$

$R = 1$

$L = .5\ \mu H$

$C = 15\ nF$

$C_e = 10\ PF$

$t = 1\ ns.$

Solution:

First each element of the circuit is written in the Laplace transformation form, which is replaced in the circuit and then the voltage is found in s domain.

In Laplace transformation form each element is written as follows:

$$C \longrightarrow 1/sC$$

$$L \longrightarrow sL$$

$$R \longrightarrow R$$

$$C_e \longrightarrow 1/sC_e$$

Let the following symbols be assigned for the sake of convenience:

$$A = (R + sL + 1/sC)$$

$$B = (1/sC_e)$$

then voltage $e_2(t)$ is found as follows:

$$e_2(s) = e_1(s)\ (B(A + B))/(AB + (A + B)^2)$$

$$= e_1(s).C(LCC_e s + RCC_e s + C + C_e)/(CC_e(LCs^2 + RC + 1) + (LCC_e s^2 + RCC_e s^2 + C + C_e)$$

$$e_1(s) = 1/s$$

then

$$e_2(s) = (1/s)(2.10^{17}s^2 + 4.10^{23}s + 4.10^{34})/(s^4 + 4.10^6 s^3 + 6.0027.10^{17}s^2 + 12.10^{23}s + 4.10^{34})$$

The roots of the polynomial of denominator are found by digital computer. Thus $e_2(s)$ can be written in the following form:

$$e_2(s) = 1/s - .3619499/(s + 999406 - j2.763.10^8) - .3619599/(s + 999406 + j2.763.10^8) - .131522/(s + 1.00059.10^6 - j7.2382.10^8) - .131522/(s + 1.00059.10^6 + 7.2382.10^8).$$

By inverse Laplace transformation and having:

$$e^{\pm ja} = \mathrm{Cos}\ a \pm j\mathrm{Sin}\ a$$

$e_2(s)$ can be written in t domain as follows:

$$e_2(t) = 1 - .724e^{-999406t}\mathrm{Cos}(2.763.10^8 t) - .276e^{-10^6 t}\mathrm{Cos}(7.2382.10^8 t) \qquad (73)$$

In order to be able to compare the Laplace transformation technique with the digital computer method by use of trapezoidal rule of integration, the above example has been solved by both techniques.

Result of Laplace Transformation Technique

TIME	VOLTAGE AT NODE 2
0	0
1.E-9	0.097563
2.E-9	0.351156
3.E-9	0.667806
4.E-9	0.942877
5.E-9	1.10838
6.E-9	1.16165
7.E-9	1.16043
8.E-9	1.1875
9.E-9	1.30286
1.E-8	1.50823
1.1E-8	1.74178
1.2E-8	1.90613
1.3E-8	1.91578
1.4E-8	1.74019
1.5E-8	1.42101
1.6E-8	1.05462
1.7E-8	0.748648
1.8E-8	0.573778
1.9E-8	0.534704
2.E-8	0.574165
2.1E-8	0.607547
2.2E-8	0.570199
2.3E-8	0.453382
2.4E-8	0.310584
2.5E-8	0.230949
2.6E-8	0.293178
2.7E-8	0.523126
2.8E-8	0.876582
2.9E-8	1.25612
3.E-8	1.55393
3.1E-8	1.69963
3.2E-8	1.68944
3.3E-8	1.5828
3.4E-8	1.46836
3.5E-8	1.41692
3.6E-8	1.44501
3.7E-8	1.50745
3.8E-8	1.52252
3.9E-8	1.4168
4.E-8	1.16697
4.1E-8	0.817207
4.2E-8	0.463112
4.3E-8	0.209731
4.4E-8	0.124401

TIME	VOLTAGE AT NODE 2
4.5E-8	0.207565
4.6E-8	0.395829
4.7E-8	0.595426
4.8E-8	0.729149
4.9E-8	0.773219
5.E-8	0.765787
5.1E-8	0.783112
5.2E-8	0.895772
5.3E-8	1.1273
5.4E-8	1.43639
5.5E-8	1.73209
5.6E-8	1.91461
5.7E-8	1.92167
5.8E-8	1.75754
5.9E-8	1.49016
6.E-8	1.21822
6.1E-8	1.02431
6.2E-8	0.937728
6.3E-8	0.925065
6.4E-8	0.912868
6.5E-8	0.830571
6.6E-8	0.651639
6.7E-8	0.411906
6.8E-8	0.195555
6.9E-8	9.55559E-2
7.E-8	0.16819
7.1E-8	0.404405
7.2E-8	0.732339
7.3E-8	1.04986
7.4E-8	1.27103
7.5E-8	1.36357
7.6E-8	1.35896
7.7E-8	1.33069
7.8E-8	1.35196
7.9E-8	1.4546
8.E-8	1.60999
8.1E-8	1.74183
8.2E-8	1.76437
8.3E-8	1.62691
8.4E-8	1.3422
8.5E-8	0.984221
8.6E-8	0.656235
8.7E-8	0.44482
8.8E-8	0.382401
8.9E-8	0.436666
9.E-8	0.531656
9.1E-8	0.589552
9.2E-8	0.571909

TIME	VOLTAGE AT NODE 2
9.3E-8	0.499518
9.4E-8	0.440934
9.5E-8	0.475559
9.6E-8	0.650015
9.7E-8	0.949963
9.8E-8	1.30183
9.9E-8	1.60396
1.E-7	1.7719
0.000001	0.705038
1.001E-6	0.776129
1.002E-6	0.846991
1.003E-6	0.890334
1.004E-6	0.906583
1.005E-6	0.895383
1.006E-6	0.904982
1.007E-6	0.95495
1.008E-6	1.05107
1.009E-6	1.17413
1.01E-6	1.28743
1.011E-6	1.35326
1.012E-6	1.35042
1.013E-6	1.28391
1.014E-6	1.18211
1.015E-6	1.08296
1.016E-6	1.01592
1.017E-6	0.98676
1.018E-6	0.985715
1.019E-6	0.977621
1.02E-6	0.93889
1.021E-6	0.862565
1.022E-6	0.765764
1.023E-6	0.682649
1.024E-6	0.648358
1.025E-6	0.681876
1.026E-6	0.77639
1.027E-6	0.90198
1.028E-6	1.0193
1.029E-6	1.09751
1.03E-6	1.12758
1.031E-6	1.12448
1.032E-6	1.11739
1.033E-6	1.13293
1.034E-6	1.18014

TIME	VOLTAGE AT NODE 2
1.035E-6	1.24466
1.036E-6	1.29535
1.037E-6	1.29978
1.038E-6	1.24106
1.039E-6	1.12742
1.04E-6	0.989623
1.041E-6	0.867555
1.042E-6	0.792412
1.043E-6	0.773374
1.044E-6	0.795196
1.045E-6	0.827755
1.046E-6	0.842603
1.047E-6	0.828125
1.048E-6	0.795623
1.049E-6	0.773289
1.05E-6	0.791137
1.051E-6	0.864519
1.052E-6	0.984606
1.053E-6	1.12073
1.054E-6	1.23353
1.055E-6	1.29254
1.056E-6	1.28941
1.057E-6	1.24051
1.058E-6	1.17755
1.059E-6	1.13114
1.06E-6	1.11557
1.061E-6	1.12236
1.062E-6	1.12577
1.063E-6	1.09751
1.064E-6	1.02284
1.065E-6	0.910201
1.066E-6	0.789117
1.067E-6	0.697393
1.068E-6	0.663904
1.069E-6	0.695414
1.07E-6	0.773973
1.071E-6	0.866097
1.072E-6	0.939099
1.073E-6	0.976432
1.074E-6	0.984413
1.075E-6	0.987112
1.076E-6	1.01214
1.077E-6	1.07459
1.078E-6	1.16743
1.079E-6	1.26325
1.08E-6	1.32652
1.081E-6	1.33055
1.082E-6	1.27046

Result of Digital Computer Technique

TIME	VOLTAGE AT NODE 2
0	0
1.E-9	4.55102E-2
2.E-9	0.211663
3.E-9	0.487472
4.E-9	0.78475
5.E-9	1.02155
6.E-9	1.15587
7.E-9	1.19991
8.E-9	1.20881
9.E-9	1.24933
1.E-8	1.3637
1.1E-8	1.54554
1.2E-8	1.7394
1.3E-8	1.86444
1.4E-8	1.85118
1.5E-8	1.67461
1.6E-8	1.36804
1.7E-8	1.01127
1.8E-8	0.698313
1.9E-8	0.499197
2.E-8	0.433227
2.1E-8	0.465584
2.2E-8	0.528403
2.3E-8	0.556162
2.4E-8	0.518692
2.5E-8	0.43634
2.6E-8	0.370212
2.7E-8	0.392066
2.8E-8	0.547964
2.9E-8	0.832753
3.E-8	1.18747
3.1E-8	1.52125
3.2E-8	1.74787
3.3E-8	1.8206
3.4E-8	1.74937
3.5E-8	1.59288
3.6E-8	1.42949
3.7E-8	1.32049
3.8E-8	1.28294
3.9E-8	1.28434
4.E-8	1.26154
4.1E-8	1.15474
4.2E-8	0.940405
4.3E-8	0.64762
4.4E-8	0.349787

TIME	VOLTAGE AT NODE 2
4.5E-8	0.135237
4.6E-8	0.070024
4.7E-8	0.169845
4.8E-8	0.393901
4.9E-8	0.663405
5.E-8	0.896183
5.1E-8	1.04147
5.2E-8	1.09906
5.3E-8	1.11425
5.4E-8	1.15144
5.5E-8	1.25893
5.6E-8	1.44168
5.7E-8	1.65504
5.8E-8	1.82249
5.9E-8	1.86948
6.E-8	1.75774
6.1E-8	1.50421
6.2E-8	1.17581
6.3E-8	0.86214
6.4E-8	0.638779
6.5E-8	0.537508
6.6E-8	0.537069
6.7E-8	0.578177
6.8E-8	0.595449
6.9E-8	0.551115
7.E-8	0.45465
7.1E-8	0.359106
7.2E-8	0.335825
7.3E-8	0.439324
7.4E-8	0.678778
7.5E-8	1.00953
7.6E-8	1.34885
7.7E-8	1.60899
7.8E-8	1.7326
7.9E-8	1.71449
8.E-8	1.60011
8.1E-8	1.46167
8.2E-8	1.36329
8.3E-8	1.33138
8.4E-8	1.34397
8.5E-8	1.34372
8.6E-8	1.26841
8.7E-8	1.08443
8.8E-8	0.807453
8.9E-8	0.500316
9.E-8	0.248756
9.1E-8	0.125928

TIME	VOLTAGE AT NODE 2
9.2E-8	0.161782
9.3E-8	0.331253
9.4E-8	0.566532
9.5E-8	0.787665
9.6E-8	0.937359
9.7E-8	1.00404
9.8E-8	1.02279
9.9E-8	1.05407
1.E-7	1.15053

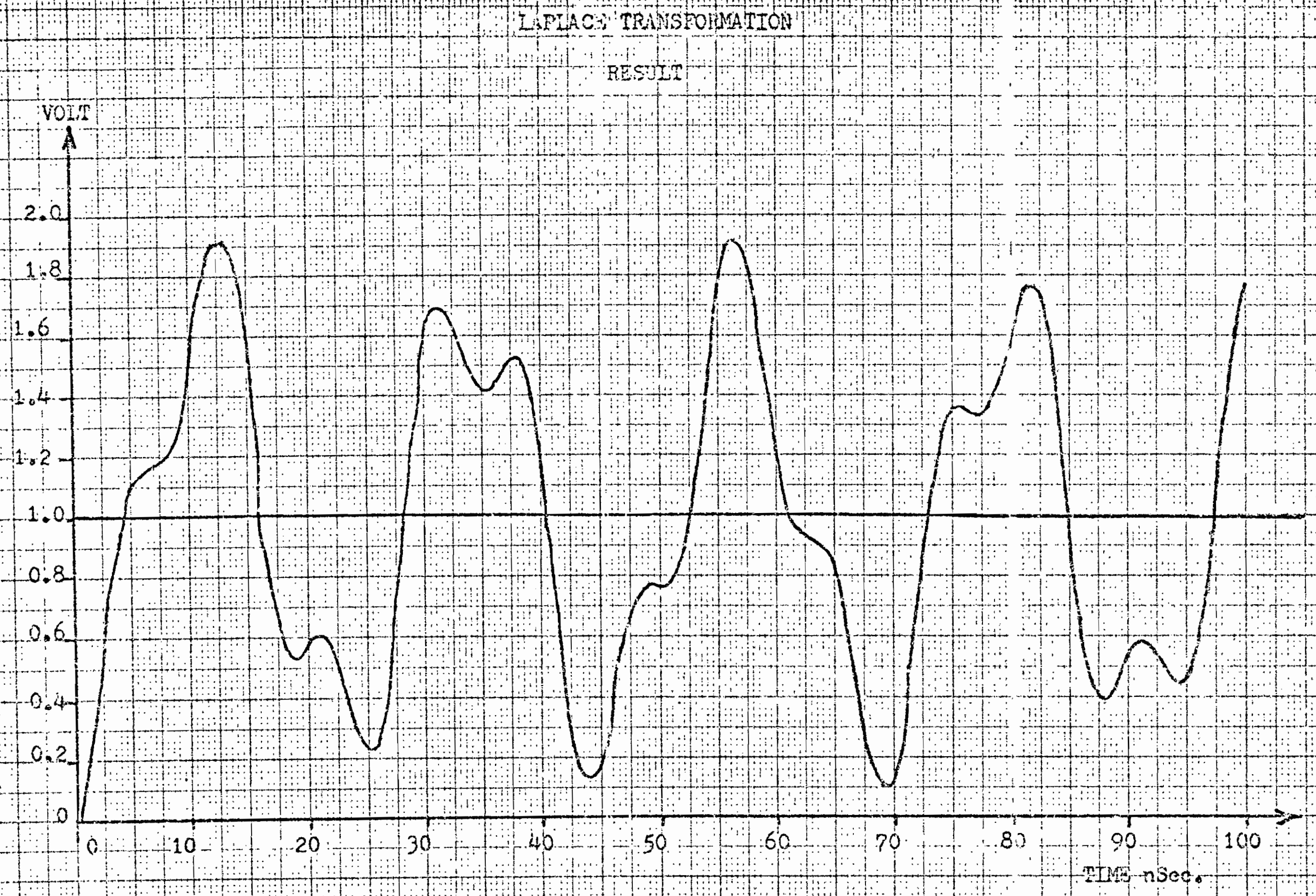

LAPLACE TRANSFORMATION
RESULT
VOLT
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
0
10
20
30
40
50
60
70
80
90
100
TIME nSec.

Fig. (47)

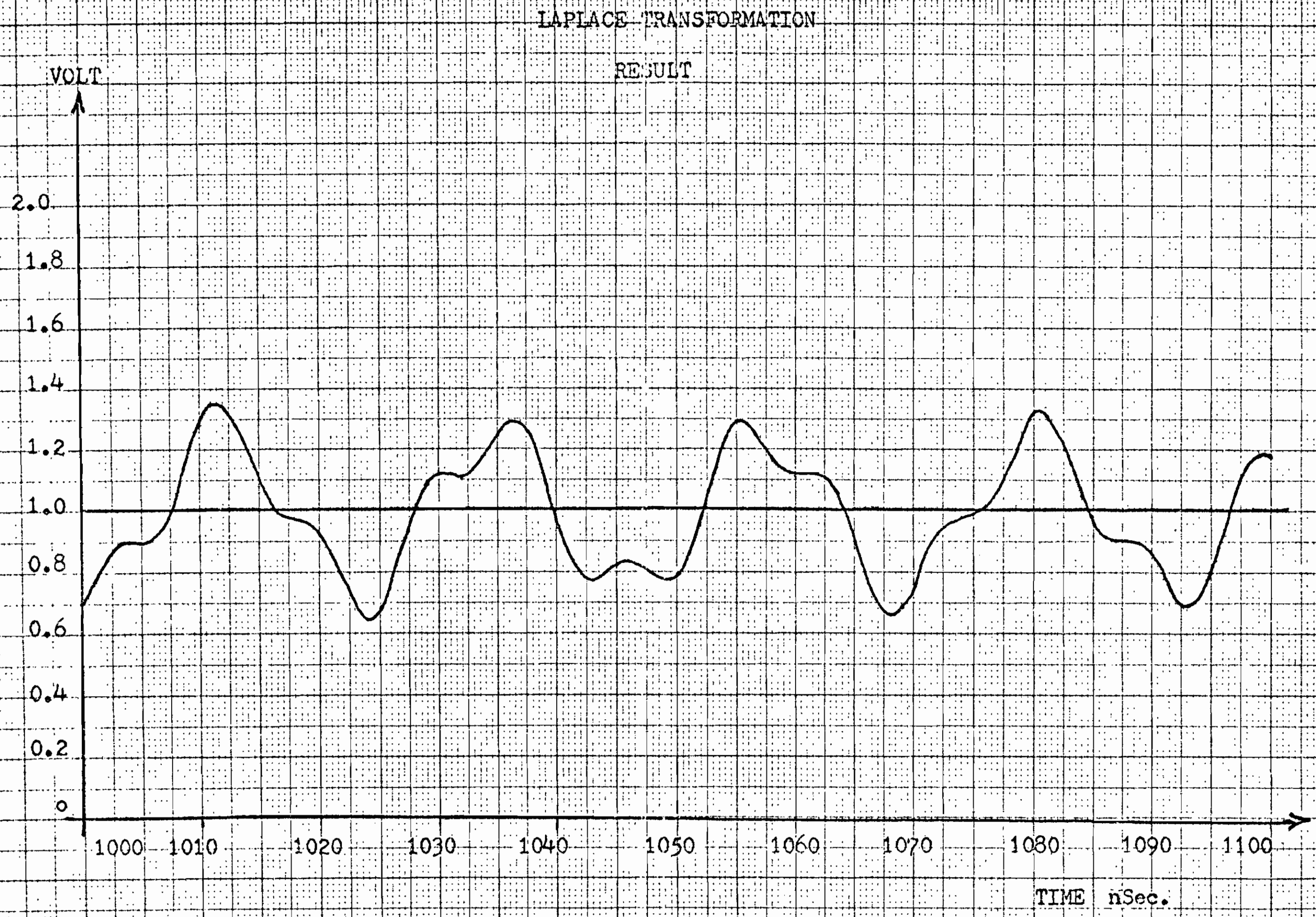

LAPLACE TRANSFORMATION
RESULT
VOLT
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
1000
1010
1020
1030
1040
1050
1060
1070
1080
1090
1100
TIME nSec.

Fig. (48)

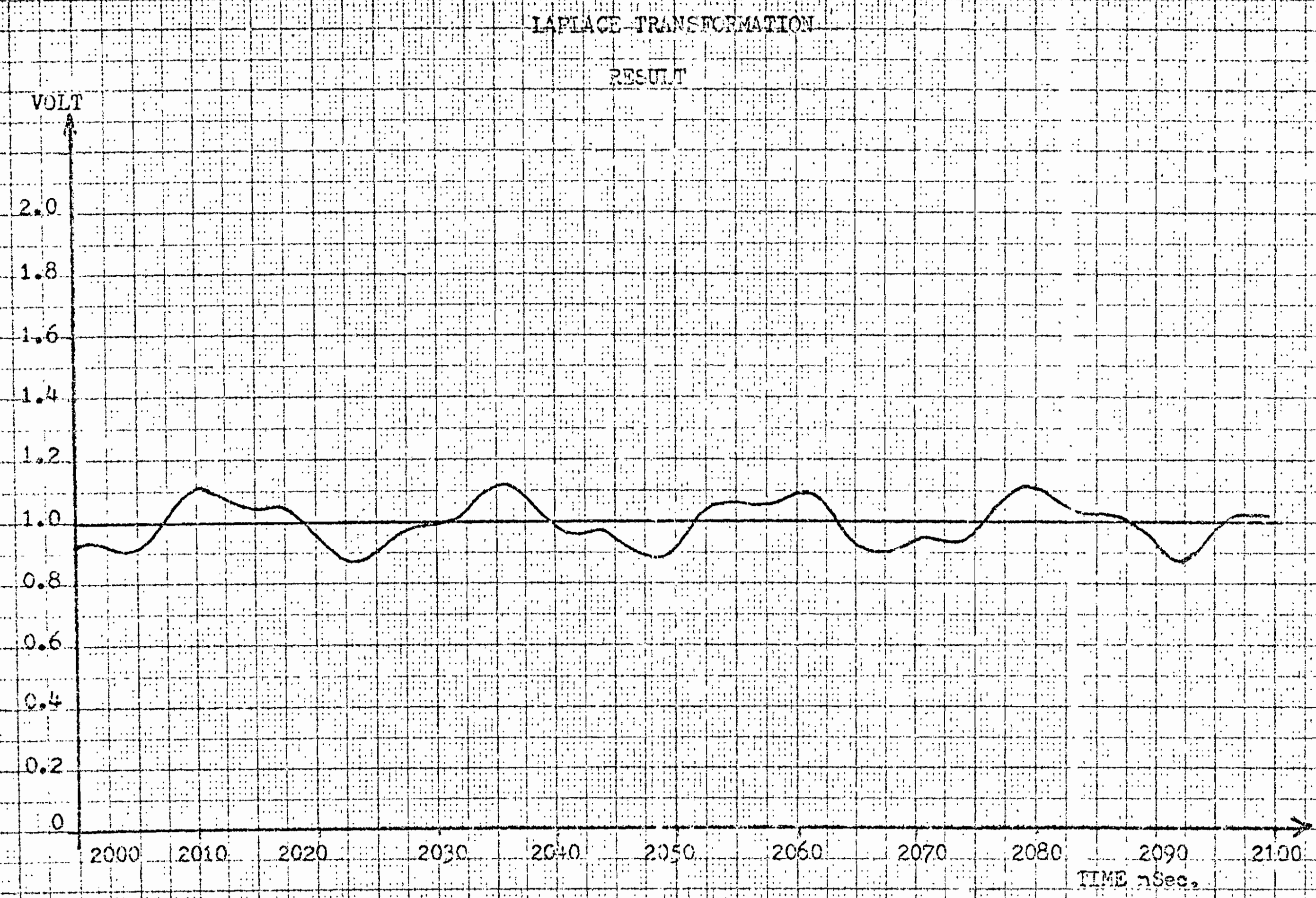

LAPLACE-TRANSFORMATION
RESULT
VOLT
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
2000
2010
2020
2030
2040
2050
2060
2070
2080
2090
2100
TIME nSec.

Fig. (49)

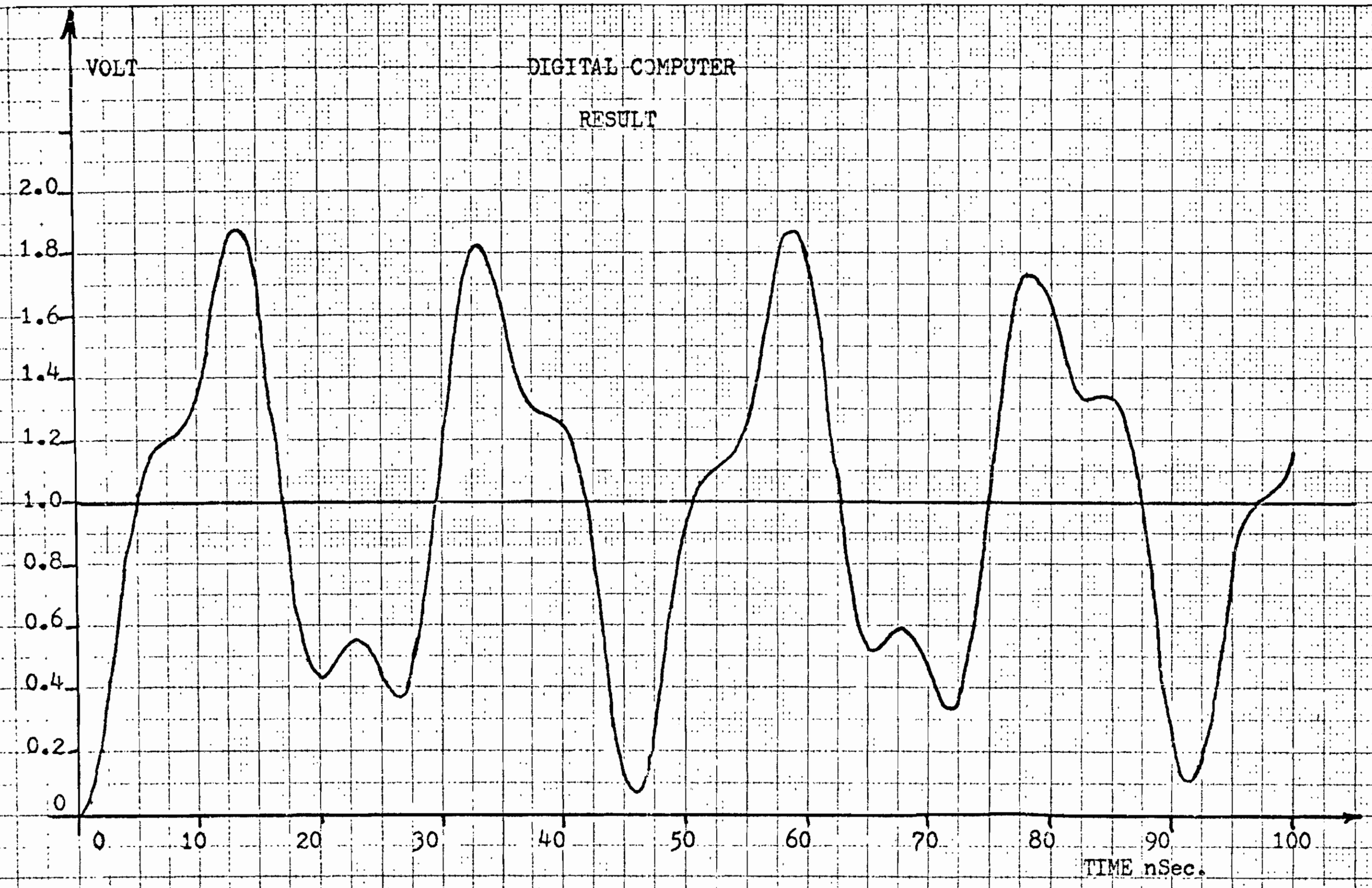

VOLT
DIGITAL COMPUTER
RESULT
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
0
10
20
30
40
50
60
70
80
90
100
TIME nSec.

Fig. (50)

CHAPTER IV

COMPARISON AND CONCLUSION

Comparison of Both Techniques

Advantages and limitations of both of these methods are considered in the following points:

Digital computer solution by the use of the trapezoidal rule of integration and the Bergeron method approach:

Advantages:

1. Once current or voltage at a node is determined, all currents and voltages at all nodes can be found simultaneously. In other words, the entire picture of the circuit with its past history can be determined at the same time.
2. Voltage or current is described by a linear equation for each element. There is no need to derive a new equation for each element in a different type of circuit.
3. Complex forcing functions such as sinusoids and exponentials can be handled exactly with no approximation.
4. There is no complex number involved and there is no need to find the roots of a polynomial.
5. This technique is accurate enough for practical purposes and it can be applied to very large power systems.

Limitations:

1. There is no exact mathematical equation describing the nature of current or voltage at a node.
2. Initial condition or the past history is always needed.

Laplace transformation technique

Advantages:

1. There is an exact mathematical equation describing the nature of current or voltage at a node.
2. There is no need for initial condition; data can be obtained at any time easily with only one equation describing the voltage or current.
3. There is no approximation involved in deriving the equations except in computing of coefficients.
4. Laplace transformation technique is analytically and practically more precise than the first method.

Limitations:

1. All or some of the current and voltages can not be found at the same time unless for each node an equation describing voltage and current has been independently derived.
2. Complex numbers are involved and the roots of a polynomial of transfer function have to be found. When the order of the polynomial is larger than 3, the roots may be found by digital computation. However, it will be sometimes impossible to find the roots by digital computation when the order of the polynomial is too large.

3. It is difficult and sometimes impossible to find the inverse matrix of an impedance matrix in a large power system.
4. Each forcing function must be transferred to Laplace form and this increases the order of the polynomial.

Conclusion

The graphical solution of the Bergeron method and its application to digital computer solution of electromagnetic transient for distributed parameters and the trapezoidal rule of integration for lumped parameters was discussed. The equations describing the relation of voltage and current on a lossless line and lumped parameters were found; consequently the equivalent impedance networks were drawn. Three examples showed the step by step digital computer solution and programming.

To compare this technique with another, the Laplace transformation method was introduced. An example was illustrated by both techniques and in conclusion the accuracy, advantages, and disadvantages of both methods were discussed.

The method of solving electromagnetic transient with the Bergeron method approach for distributed parameters and the trapezoidal rule of integration for lumped parameters is very efficient and capable of handling very large networks. This method may of course be used with multiphase systems as well as single-phase.

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